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A Negative Binomial- Garch Model with Application to Meningitis Cases in Nigeria

Oguntade E. S., Obafaiye A. B. and Oladimeji D. M.*

Department of Statistics, Faculty of Science, University of Abuja, Nigeria

Corresponding Author: oladimejidamilare@yahoo.com

ABSTRACT

Meningitis, a contagious tropical disease, disproportionately affects low-income populations with limited access to quality medical care. While previous studies have explored various methodologies for modeling meningitis cases, the issue of overdispersion remains inadequately addressed. This study applies a Negative Binomial integer GARCH model to meningitis data collected weekly from the Nigeria Centre for Disease Control between 2006 and 2021. Initial analysis revealed overdispersion in the data, justifying the use of a negative binomial GARCH model. The Augmented Dickey-Fuller test confirmed stationarity in the dataset. After evaluating multiple models, NB-INGARCH (1,2) emerged as the optimal choice based on AIC and BIC criteria. The selected model demonstrated adequacy through probability integrated transform histograms and calibration plots. This research showcases the effectiveness of NB-INGARCH in addressing overdispersion in time series data. To mitigate meningitis cases, we recommend promoting modern housing systems with adequate ventilation and decongestion measures in densely populated areas. Additionally, a targeted health awareness campaign on meningitis is suggested.

Keywords: Negative Binomial, Generalized Autoregressive Conditional Heteroscedasticity, Meningitis, Nigeria.

INTRODUCTION

Meningitis is a global phenomenon which is a common bacterial or viral disease in the tropics especially in low- and medium-income countries. It remains a major public health challenge mainly in the semi-arid areas of sub-Saharan African region (Centre for Disease Control (CDC),2024). Meningitis is a bacteria or viral infection of the brain and spinal cord fluid. It causes an inflammation of the protective membrane of brain and spinal cord. The potential causes of Meningitis include but not limited to cancer, adverse effects of drugs, injuries and other forms of infections not properly treated (CDC,2024). Meningitis caused by bacteria is the deadliest form of Meningitis which usually result into death or left permanent disabilities. Such disabilities include brain damage, learning impairment and sometimes result into memory loss. While viral meningitis is less serious than bacteria, fungal meningitis is rare and common only among those with certain health conditions like HIV, cancer and diabetes (WHO,2023). Risk factors of Bacterial Meningitis include age group, medical conditions (especially those with weak immune systems), group settings, and exposure to Meningitis pathogens amongst others have been identified in literature (Ahmed *et al.,* 202).

The global statistics revealed that about five million new cases were reported in 2019 which resulted to about 290,000 deaths (Ahmed *et al.,* 2021) while 2023 Meningitis reports in Nigeria between October 2022 and April 2023 revealed about 1686 cases which resulted into about 124 deaths for a case fatality ratio of 7%. These reported cases were predominant in male than the female and mostly in children aged 1 to 15 years (WHO,

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2024). Cerebrospinal Meningitis is the most common Meningitis in Nigeria with highest prevalence reported in the Northern part especially in places with high and densely populated like Kano and Jigawa State (Ahmed *et al.,* 2021). Occasional outbreak of this disease has become a usual phenomenon and usually occur at the peak of hot weather.

Numerous studies on modelling infectious diseases like meningitis were documented in literature exploring various statistical and mathematical techniques (Adekanmbi and Olaomi, 2014; Oguntade *et al.,* 2020). While a significant number of studies identified and explored disease modelling from a mathematical technique (Asamoah *et al.*, 2018) others approached disease modelling using various statistical methods (Ahmed *et al.,* 2021). For instance, Asamoah *et al.* (2018) approached modelling of meningitis from a nonlinear deterministic perspective with time-dependent controls to describe the dynamics of bacterial meningitis in a population while Adekanmbi and Olaomi (2014) constructed a time series model for meningitis based on the methodology espoused in Box and Jenkins. Aaishah *et al., (*2020) constructed a negative binomial integer generalized autoregressive conditional heteroscedasticity for modelling Asthma in Malaysia. Oguntade *et al.* (2020) used a Poisson and a Negative Binomial regression model to explore the effect of weather factors on malaria occurrence in Nigeria.

This present study introduced a Negative Binomial Integer Value Autoregressive Generalized conditional heteroscedasticity (NB-INGARCH) model to model meningitis cases in Nigeria. The choice of this model was based on its robustness in handling discrete data and data that are over-dispersed in nature. The NB-INGARCH models were well explored elsewhere for disease modelling (Asamoah *et al.*, 2018).

MATERIALS AND METHODS

Study Design

This study was a retrospective study on meningitis cases in Nigeria. Due to the yearly occurrence of meningitis, modeling meningitis becomes a necessity. This research was carried out using a Negative Binomial Autoregressive Generalized conditional heteroscedasticity model.

Data Source

The data collected for this study were meningitis cases on a weekly basis from the update released by the Nigerian Centre for Disease Control (NCDC). It covers the period from 2006 to 2021. (https://covid19.ncdc.gov.ng)

A Negative Binomial Distribution

The negative binomial distribution, like the normal distribution, arises from a mathematical formula in Equation 1. It is commonly used to describe the distribution of count data, such as the numbers of parasites in blood specimens, number of daily occurrences of a disease like cases of meningitis. Also, like the normal distribution, it has two parameters, which includes shape parameter (k). The probability mass function of the negative binomial distribution is

$$f(k, r, p) = \Pr(X = k) = \binom{k+r-1}{r-1} (1-p)^k p^r \qquad (1)$$

where r is the number of successes, k is the number of failures, and P is the probability of success.

Negative Binomial Integer Generalized Autoregressive Conditional Heteroscedasticity (NB-INGARCH)

Considering the Negative Binomial INGARCH (p, q) given in Equation 2.

$$Y_t \mid F_{t-1} \sim \mathrm{NegBin}(\lambda_{t,} \phi) \quad (1)$$

$$g(\lambda_t) = \beta_0 + \sum_{k=1}^{p} \beta_k g\left(Y_{t-ik}\right) + \sum_{i=1}^{p} \alpha_i g\left(\lambda_{t-ji}\right) + \eta^T X_t + \sum_{m=1}^{a} w_m \, \delta_m^{t-\tau_m} I(t \geq \tau_m) \quad (2)$$

Model efficiency

The model was evaluated using Pearson residuals, the probability integral transform (PIT) histogram, Akaike's Information Criteria (AIC) and the Bayesian Information Criteria (BIC). Pearson residuals was explained by Equation (3):

$$\tau_t^p = \frac{(yt - \hat{\lambda})}{\sqrt{\hat{\lambda_t} + \lambda_t^2 \, \sigma^2.}} \quad (3)$$

For t = 1,…..,n the empirical autocorrelation function (ACF) of this residual is valuable for diagnosing serial dependency. A plot of residuals versus time can reveal changes of the data generating process over time. Meanwhile, to access the probabilistic calibration of the predictive distribution, the probability integral transform (PIT) was used. PIT is given by Equation (4):

$$F_t(u|y) = \begin{cases} 0, & u \leq p_t(y-1) \\ \frac{u - p_t(y-1)}{p_t(y) - p_t(y-1)}, & p_t(y-1) < u < p_t(y). \\ 1, & u \geq p_t(y) \end{cases} \quad (4)$$

The mean PIT is the given by:

$$\bar{F}(u) = \frac{1}{n} \sum_{t=1}^{n} F_t(u|y_t), \; 0 \leq u \leq 1..$$

Order to examine whether $\bar{F}$ is the cumulative distribution function of a uniform distribution or not. The following equation suggest plotting a histogram with H bins, where bin h has the height of

$$f_t = \bar{F}(h|H) - \bar{F}\left(\frac{h-1}{H}\right), h, ..., H. \quad (5)$$

A U-shape the histogram designates an under-dispersion of the predictive distribution whereas an upside-down U-shape of histogram designate an over-dispersion. Other popular tools of the model selection criteria are the log-likelihood value, Akaike's information criteria (AIC) and the Bayesian information criteria (BIC). The model with the lowest value of the respective information criterion is preferable.

The AIC and BIC is obtained as follow:

$$AIC = 2\hat{\ell}\left(\widehat{\theta,} \; \sigma^2\right) + 2df \quad (6)$$

$$BIC = 2\hat{\ell}\left(\widehat{\theta,} \; \sigma^2\right) + \log\left(\eta_{c\int\int}\right) df. \quad (7)$$

RESULTS AND DISCUSSION

Descriptive Statistics of Meningitis Cases

Table 1 shows the summary statistics of weekly Meningitis cases. The average number of weekly cases was 24. The maximum number of cases was 835. The median value was 5.0. Since variance was larger than the mean, it indicates the presence of overdispersion in the series. Figure 1 presents the time series plot for weekly meningitis cases in Nigeria from 2006 to 2021.

Table 1: Descriptive Statistics of Weekly Meningitis Cases in Nigeria.

Median	Mean	Max.	Variance
5.00	24.20	835.00	7726.06

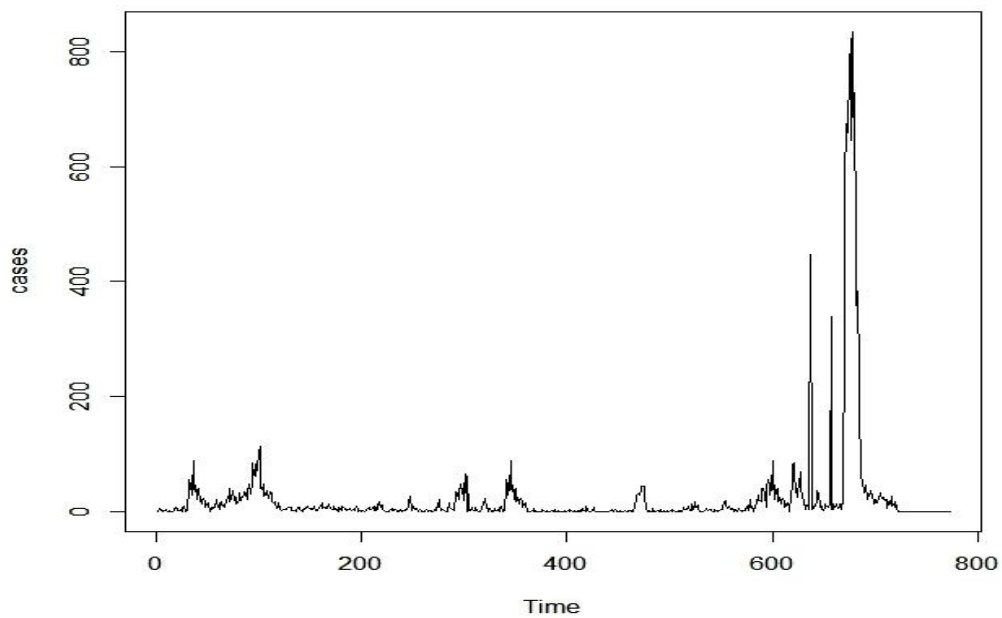

Figure 1: Time Series Plot for Weekly Meningitis Cases in Nigeria from 2006 to 2021.

As observed from Figure 1, the series plot shows the weekly number of Meningitis cases in Nigeria. There was a spike from the plot in week 10, 2020. This spike represents the highest number of meningitis cases for one consecutive year.

Unit Root Test and Marginal Calibration Plot of Meningitis Cases

Table 2 displays the Augmented Dickey-Fuller (ADF) test results of the meningitis cases. The ADF-test returns a p-value (0.021), this implies that the null hypothesis of non-stationary was rejected thereby accepting the alternative hypothesis.

Table 2: Unit Root Test for Meningitis Cases in Nigeria

	Dickey-fuller	*p*-value
I(0)	- 8.2463	0.021

Probability value: P-value, Differencing: I

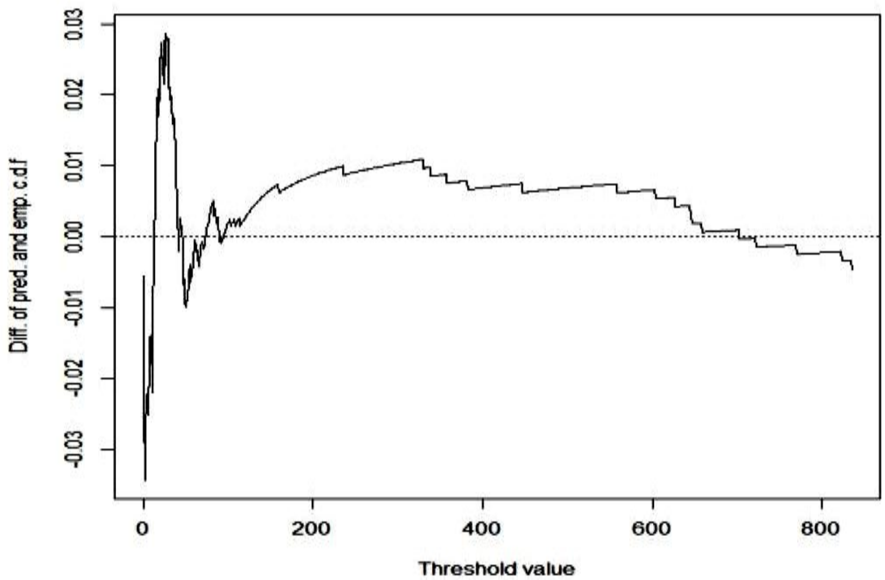

Figure 2: Marginal Calibration Plot of Meningitis Cases in Nigeria.

The marginal calibration plot, shown in Figure 2 is said not to be adequate for a Negative Binomial model except if and only if the PIT plot is uniform.

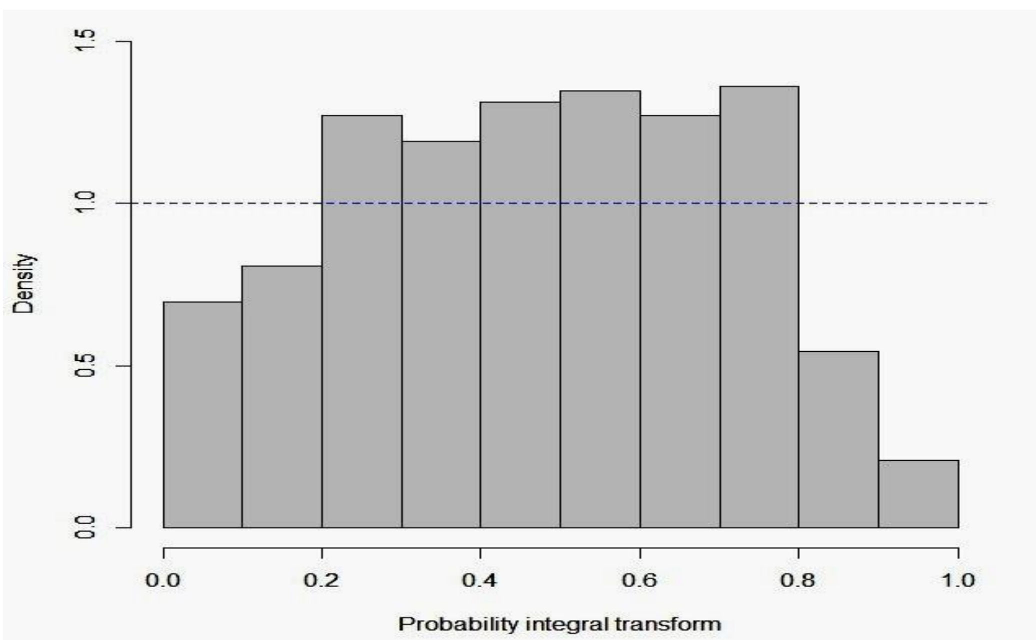

Figure 3: Probability Integral Transform of Meningitis Cases in Nigeria

The PIT histogram of the Negative Binomial distribution tends to approach uniformity better. Thereby concluding that the probabilistic calibration of the Negative Binomial model in Figure 2 was satisfactory and adequate for the model.

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Parameters Estimate of Meningitis Cases and Model Selection Criteria

Table 3 shows the Standard errors and p-values obtained by normal approximation. The intercept β_0 estimate of 1.914, with standard error of 0.3277and p-value of 0.004 were obtained. The estimated β_1 was 0.588, standard error of 0.1016 and p-value of 0.0180. The estimated β_2 was 0.498, standard error of 0.1225 and p-value of 0.0220. The estimate for α was -0.759, standard error was 0.2018 and p value of 0.0101. The result revealed an estimate of an over dispersion coefficient sigmasq as 0.629.

Table 3: Parameters Estimate of Meningitis Cases in Nigeria (NB-INGARCH 1,2)

Coefficients:	(Intercept)	β1	β2	α	sigmasq
Estimate	1.914	0.588	0.498	-0.759	0.629
Std.Error	0.3277	0.1016	0.1225	0.2018	
P value	0.0004	0.0180	0.0220	0.0101	

The fitted model (NB-INGARCH 1,2) is given in the Equation 5.

$$Y_t|F_{t-1} \sim \mathrm{NegBin}(\lambda_t, 0.629) \quad (8)$$

$\log(\lambda_t) = 1.914 + 0.588og(Y_{t-1} + 1) + 0.498\log Y_{t-2} - 0.759\log(\lambda_{t-1},)$, where $t = 1,\ldots,773$.

Table 4 is the link function (log) Table. The distribution family is negative binomial with over dispersion coefficient sigmasq. Number of coefficients was 6 (intercept, beta1, beta 2, alpha, eta_1 and sigmasq. Log likelihood calculated was -2404.572. Akaike's information criteria was 4821.144, Bayesian information Criteria was estimated as 4849.045.

Table 4: Estimate of Model Efficiency Criteria

Distribution family: nbinom (with overdispersion coefficient 'sigmasq')
Number of coefficients: 4
Log-likelihood: -2404.572
AIC: 4821.144
BIC: 4849.045

Table 5 shows the predictive model that was used for parameters estimation of all the selected model candidate. The model with lowest AIC or BIC value was preferred. From the results presented in Table 5, NB-INGARCH (1,2) has the lowest value of AIC and BIC as compared with other candidate models; NB-INGARCH (1,1), NB-INGARCH (1,3) and NB-INGARCH (1,4).

Table 5: List of Candidate Models

Models	AIC	BIC
NB INGARCH(1,1)	4945.43	4968.682
NB INGARCH(1,2)	4821.144	4849.085
NB INGARCH(1,3)	4827.685	4860.237
NB INGARCH(1,4)	4829.24	4866.44

The model comparison in Table 5 shows that NB INGARCH (1,2) was the most adequate model with the lowest AIC and BIC.

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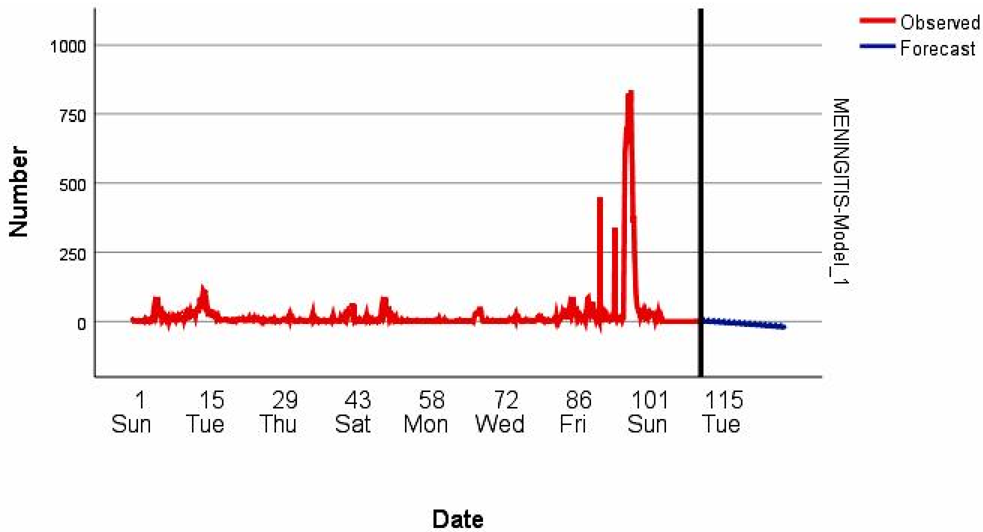

Figure 4: Observe and Forecast Plot for Weekly Meningitis Cases in Nigeria

Figure 4 shows the trend line of the time series for the data set for the model and predicted values based on the formulated model. The red line was the model data set and the light blue line was for the trend of the predicted values generated by the model. This plot verified whether the predictions were close to real values. Using the Plot, the prediction detected a decrease in forecast although, similar to the same magnitude as the real-time series. Despite that, all predicted values showed consecutive slight decrease in meningitis cases which, corresponds almost perfectly with reality.

DISCUSSION

The preliminary analysis revealed that there was presence of over dispersion in the meningitis cases (the variance is greater than the mean). This agrees with a related work on Covid 19 cases and malaria cases in Nigeria (Atigi *et al*.,2022; Oguntade *et al,* 2020). The authors observed over-dispersion problem, thus adopted a Negative Binomial method due to its capabilities in handling problem such as over dispersion.

This study demonstrated that NB-INGARCH can also be used in Meningitis modelling to forecast future trend of the disease like Asthma modelling reported somewhere in Johor Bahru *(*Aaishah *et al., 2020)*. The authors opined those results based on NB-INGARCH with identity and log link function is adequate in representing the asthma data than P-INGARCH model. The current study also identified NB INGARCH (1,2) as the most adequate model with the lowest AIC and BIC values. This agrees with Jens (2021) The author identified NB-INGARCH (1,1) as the best of the candidate models when comparing AIC and some scoring rules. Model adequacy of the NB-INGARCH (1,1) checked using PIT histograms.

The probability Integral Transform as shown in Figure 3 displayed an upside-down U shape which indicate that, the Negative Binomial distribution tends to approach uniformity better. Thereby concluding that the probabilistic calibration of the Negative Binomial model in Figure 2 is satisfactory. The PIT addressed the level of dispersion between the model and the data - like the standardized Pearson residuals. This result of

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this study agrees with Jens (2021). The plausible reason for the reduction in the forecast values of meningitis may likely be because of vaccination. The more the number of people that get vaccinated the less the number of disease cases or outbreaks recorded. This result of this study is in line with submission of (Makeri et al., 2023) on diphtheria outbreak in Nigeria.

CONCLUSION

A negative Binomial integer generalized autoregressive conditional heteroscedasticity analysis approach was applied to model the number of weekly meningitis cases in Nigeria for the period of 2006 to 2021. The negative binomial INGARCH model performed well in dealing with discrete data and data that are over disperse in nature. The results revealed that NB-INGARCH (1,2) was the best model in terms of AIC and other scoring metrics. In terms of model adequacy, NB-INGARCH (1,2) was close to adequate for the data based on the PIT histograms plot. Forecast also showed a decrease in the expected cases of meningitis in the nearest future. Therefore, emphasis should be given to having good and modern housing systems especially in places with extreme weather conditions to allow for good ventilation and decongestion of highly populated areas of various towns and cities. Also, creating a health awareness campaign on Meningitis is recommended.

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