Bima Journal of Science and Technology, Vol. 8(2B) July, 2024 ISSN: 2536-6041

DOI: 10.56892/bima.v8i2B.704

Effects of Modified Magnetic Field on Unsteady Flow of Viscoelastic Stratified Fluid Between Two Parallel Inclined Riga Plates with Viscous Dissipation

Lawal Oluwafemi Waheed

Department of Mathematics, Tai Solarin University of Education Ijagun, Ogun Sate, Nigeria

Corresponding Author: waheedlawal207@yahoo.com

ABSTRACT

The study in this paper investigates the effect of modified magnetic field on unsteady flow of viscoelastic stratified fluid between two parallel Riga plates inclined at an angle θ. The magnetic field is generated by the upper and lower Riga plates which are kept stationary. These Riga plates are kept at different temperature which decay with time with fluid density and viscosity are considered to be variable among different layers within the medium. The non-linear PDEs that govern the flow problem are transformed into non-dimensional PDEs and solved numerically using explicit finite difference scheme. Maple 23 is used to implement and simulate these schemes and the result are analyzed and presented inform of graphs. It is established that the effect of Reynolds number, viscoelastic parameter, stratification parameter and Prandtl number on the velocity and temperature distribution are subdued by modified magnetic field.

Keywords: Riga plates, Modified magnetic field, Viscoelastic fluid, Finite difference scheme, Stratification parameter, Lorentz force.

INTRODUCTION

Viscoelastic fluids as one of the types of non-Newtonian fluid exhibit both the properties of viscous and elastic component. In fact, they are the mixture of a solvent and some polymer. Fluids such as paints, cement slurries, mayonnaise, creams, butter, paste, some biological fluids and others from the food and chemical industry are examples of viscoelastic fluid. A few applications including viscoelastic liquid are seen in miniature apportioning of bioactive liquids through high throughput infusion gadgets, production of cell connection destinations, platforms for tissue designing, coatings and medication conveyance frameworks for controlled drug discharge. In flat or downhill sections of a horizontal channel, stratified flow typically occurs at low flow rates, whereas in uphill sections, the interface between the fluids becomes mixed and irregular as flow rate increases.

The Riga plate, on the other hand, produces a plane surface rather than polarity and magnetization by combining electrodes and permanent magnets. This thus creates the electromagnetic hydrodynamic liquid way of behaving that is utilized generally in modern cycles with liquid stream undertakings. In other to achieve an effective and better fluid flow control, the external magnetic or electric field can be applied when the motion of the fluid is low. This concept was first introduced by Gallites and Lilausis (1961) to create an executed magnetic field over electric fields and thus generate Lorentz force to control the flow of fluid. As a result, the applications of magneto-aerodynamics, civil engineering, mechanical engineering, chemical engineering, gas dust or fumes, biomechanics, groundwater rectification, and oil all rely heavily on the Riga plate.

Recently, it is noticed that many researchers have been showed interest in contributing their views on fluid flow through a Riga Plate.

Prakash et. al. (2012) examines the impacts of heat transfer in MHD stream of viscoelastic stratified fluid in permeable medium on an equal plate channel inclined at an angle θ. The impacts of different parameters particularly inclination factor on temperature and speed field of the fluid are shown through diagrams and discussed numerically.

Hydromagnetic viscoelastic fluid stream between two flat endless equal permeable plates with time changing sinusoidal pressure gradient and magnetic field has been researched by Dash and Ojha (2018). By embedding the channel in a porous medium, they were able to demonstrate that a low-frequency oscillating pressure gradient prevents back flow and reduces skin frictions, while a magnetic field and elasticity slow the fluid flow.

A new approach was used by Sattar and Abeer (2019) to analyze the axisymmetric viscoelastic squeezed flow between two parallel plates. The coefficients of powers series produced by integrating nth-order differential equations with known data are the primary determinant of this new method. With this, they acquired a logical approximate solution for the squeezing flow between two equal plates.

Mollah (2019) laid out the mathematical investigation of EMHD laminar flow of Bingham fluid streaming between two Riga plates produced into the thermal radiation effect. Through tables and graphs, he was able to demonstrate how certain parameters affected the flow pattern and the local shear stress, including the local Nusselt number.

Krishna et al. (2020) conducted an unsteady free convective magneto-hydrodynamic flow with Hall and ion slip current effects through a rotating, accelerated inclined plate surrounded by a porous medium with the effect of an inclined angle and a shift in reference frame. Analytically, they used the Laplace transform to solve these issues.

Islam and Nasrin (2021) broke down the unsteady laminar progression of intensity adaptable dusty fluid between two equal Riga plates by presenting a uniform Lorentz force through a Riga plate and applied a consistent tension slope to the fluid. They talked about the impacts of actual boundaries on the velocity and temperature profile as well as the shear pressure and Nusselt number of clean fluid and dust particle.

Nasrin et al. (2021) researches the unstable Couette flow with Hall and ion-slip current impacts between two Riga plates and present their mathematical solutions utilizing explicit finite difference methods. Their findings captured the effect that the Riga plates had on the flow profiles' modified Hartmann number.

Elshabrawy et al. (2023) shows logical solution of thermal impact on unstable viscoelastic dusty fluid between two equal plates within the sight of various pressure gradients with an intensity source or an intensity sink. Their outcomes outline the impacts of the thermal diffusion parameter and synthetic response boundary on fluid and residue particles' velocity.

Getting motivation from the above researches, the present study investigates the stratification effect of unsteady flow of viscoelastic fluid between two vertical Riga plates with viscous dissipation. The explicit finite difference technique will be adopted as a core tool to solve the system of partial differential equation involved. The schemes obtained will be implemented on MAPLE 23 to calculate require results in order to show the effect of different parameters on the velocity and temperature profile.

DOI: 10.56892/bima.v8i2B.704

Problem Formulation

We considered an unsteady fully developed flow of an incompressible stratified viscoelastic fluid over a parallel Riga plate's detached appart by $2h$ and horizontally inclined by an angle θ. The two Riga plates are kept stationary with upper plates at $y = h$ and lower plate at $y = -h$. The flow direction is vertical with width of the plates corresponding to the $xz-$ plane.

Assumptions

The following assumptions are made in formulating the governing equations

i. Initially, the fluid is kept stationary (i.e there is no flow) and the Riga plates are two different temperatures ($t = 0,\ T = T_w,$ at $y = -h$ and $t > 0,\ T = T_1$ at $y = h$).

ii. Later the fluid is kept in motion by pressure gradient force $\frac{\partial P}{\partial x}$ applied to the fluid in x-direction

iii. An unchanging magnetic force is produced by the Riga plate

iv. The fluid considered in this paper is finitely conducting so that magnetic dissipation due to the existence of external magnetic field by Riga plate is insignificant

v. The induced magnetic field is not measured due to small value of magnetic Reynolds's number.

vi. The fluid density and viscosity are given as $\rho = \rho_0 e^{-s(y/t+1)}$ and $\mu = \mu_0 e^{-s(y/t+1)}$ respectively are changing along y - axis all over the plates. s is the stratification factor of the fluid while ρ_0 and μ_0 are the coefficient of viscosity and density correspondingly along the lower plate.

Riga plate

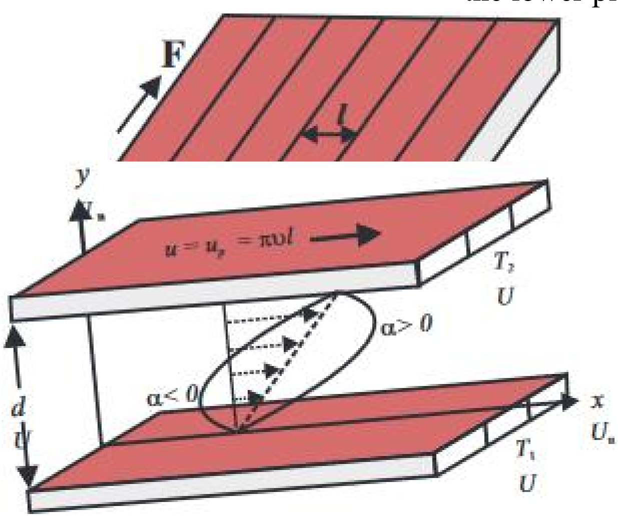


Figure 1: Physical model of the system

Because the fluid is passing through the Riga plates, the magnetic force due to the Lorentz force

$F = J \wedge B$ where $J = \sigma(E + q \wedge B)$ represent current density as explained by Ohm's law and can be neglected due to weak conductivity of viscoelastic fluid. Therefore, using the Lorentz force along x - axis is $F = \sigma(E \wedge B)$ and according to the Grinberg hypothesis, magnetic force is

DOI: 10.56892/bima.v8i2B.704

defined $F = \left(\frac{\pi}{8} J_0 M_0 e^{-\frac{\pi}{a} y} \right)$. By considering the above assumptions, the equation the govern the fluid flow are presented as:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial x} - \frac{\mu_1}{\rho}\left(\frac{s}{h}\frac{\partial u}{\partial y} + \frac{\partial^2 u}{\partial y^2}\right) + \frac{\mu_2}{\rho}\left[\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y^2}\right)\right] + \frac{g}{\rho}\sin(\theta) + \frac{\pi}{8} J_0 M_0 e^{-\frac{\pi}{a} y} \tag{1}$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial y^2}\right) - \frac{\mu_1}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\mu_2}{\rho c_p}\left[\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial y}\right)^2\right] \tag{2}$$

$$P = \rho g\left(x\sin(\theta) - y\cos(\theta)\right) + \rho x \alpha(t) + \beta \tag{3}$$

With initial and boundary conditions

$$t \le 0: \quad u = 0,\ T = T_0 \tag{4}$$

$$t > 0: \quad u = 0,\ \ T = T_0 e^{-2st} \ \text{ at } \ y = -h \tag{5}$$

$$u = u_0 e^{-st},\ \ T = T_1 e^{-2st} \ \text{ at } \ y = h \tag{6}$$

where T_0 and T_1 are plate temperature at $y = -h$ and $y = h$ correspondingly where u and T areBthe velocity and temperature of the fluid respectively, μ_1 and μ_2 are kinematic coefficient of the fluid viscosity and viscoelasticity respectively. ρ is the fluid density, c_p is the specific heat constant pressure, k is the thermal conductivity of the fluid, θ is the angle of inclination. g is the acceleration due to gravity, P represent fluid pressure. Using the following non dimensional parameters

$$\bar{u} = \frac{u}{u_0}, \quad \bar{y} = \frac{y}{h}, \quad \bar{t} = \frac{t u_0}{h}, \quad \bar{a} = \frac{ah}{u_0^2}, \quad \bar{T} = \frac{T}{T_0} \tag{7}$$

Substituting equation (7) and (3) into Equation (1) – (6) and remove “-“ , we obtain

$$\frac{\partial u}{\partial t} = -\alpha - \frac{s}{R_e}\frac{\partial u}{\partial y} + \frac{1}{R_e}\frac{\partial^2 u}{\partial y^2} + \eta\left[\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y^2}\right)\right] + H_r e^{-y} \tag{8}$$

$$\frac{\partial T}{\partial t} = \frac{1}{P_r}\frac{\partial^2 T}{\partial y^2} - \frac{E_c}{R_e}\left(\frac{\partial u}{\partial y}\right)^2 + \eta E_c\left[\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial y}\right)^2\right] \tag{9}$$

With a reduced initial and boundary conditions;

$$t \le 0: \quad u = 0,\ T = 0 \tag{10}$$

$$t > 0: \quad u = 0,\ \ T = e^{-2st} \ \text{ at } \ y = -1 \tag{11}$$

$$u = e^{-st},\ \ T = \chi e^{-2st} \ \text{ at } \ y = 1 \tag{12}$$

where $\chi = \frac{T_1}{T_0}$ is the constant temperature

$R_e = \frac{u_0 h}{\mu_1}$ Represent Reynolds number

$\eta = \frac{-\mu_2}{h^2}$ Is the viscoelastic parameter

$H_r = \frac{h^2 J_0 M_0}{8\rho\pi u_0 \mu_1}$ is the modified Hatmann number

$P_r = \frac{\mu_1}{k}$ is the Prandth number

$E_c = \frac{\mu_0^2}{\rho c_p T_0}$ represent Ecket number

Method of solution

The non-dimensional partial differential equations (8) and (9) are solved together with the associated initial and boundary conditions (10) and (12) using explicit finite difference scheme illustrated below:

$$\frac{u_{i,j}^{n+1} - u_{i,j}^{n}}{\Delta t} = \alpha + \frac{s}{R_e}\frac{u_{i,j}^{n} - u_{i,j-1}^{n}}{\Delta y} + \frac{1}{R_e}\frac{u_{i,j+1}^{n} - 2u_{i,j}^{n} + u_{i,j-1}^{n}}{(\Delta y)^2} - \eta\left(\frac{u_{i,j+1}^{n+1} - 2u_{i,j}^{n+1} + u_{i,j-1}^{n+1}}{(\Delta y)^2}\right) + H_r e^{-y} \quad (13)$$

$$\frac{T_{i,j}^{n+1} - T_{i,j}^{n}}{\Delta t} = \frac{1}{P_r}\left[\frac{T_{i,j+1}^{n} - 2T_{i,j}^{n} + T_{i,j-1}^{n}}{(\Delta y)^2}\right] + \frac{E_c}{R_e}\left[\frac{u_{i,j}^{n} - u_{i,j-1}^{n}}{\Delta y}\right]^2 + \eta E_c\left[\frac{u_{i,j}^{n+1} - u_{i,j-1}^{n+1}}{\Delta y}\right]^2 \quad (14)$$

with boundary condition

$$u_{i,L} = 0, \qquad T_{i,L} = e^{-2st_i} \qquad \text{at } L = -1 \text{ for all } n \quad (15)$$

$$u_{i,L} = e^{-st_i}, \qquad T_{i,L} = \chi e^{-2st_i} \qquad \text{at } L = 1 \text{ for all } n \quad (16)$$

where subscript i and j are x and y respectively and super subscript n refers to time t

The length of the plates is infinitely long and the plates are at distance $h = 2$ apart with upper plate fixed at $y_{\max} = 1$ and the lower plate at $y_{\min} = -1$. The viscosities drag acting at the plates for fluid τ_f is given as $\tau_f = \left[\left(\frac{1}{R} - s\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial y}\right]$. The flow flux for the fluid through the Riga plate is presented as $\phi_f = \int_{-1}^{1} u\,dy$ and the rate of heat transfer in term of Nusselt number (N_u) at the plate is represented as $N_u = \left[\frac{\partial T}{\partial y}\right]_{y=\pm 1}$

RESULTS AND DISCUSSION

As a result of iterating the numerical scheme above using Maple 23, the effect of the relevant non-dimensional parameter on the velocity profile and temperature profile at a very high and low constant value of modify magnetic field (H_r) were presented and discussed with the aid of graph. In Figure 2, it is realized that at a very high or low H_r increase in pressure parameter (α) increases

DOI: 10.56892/bima.v8i2B.704

the velocity and temperature distribution. The reason is that increase in pressure cause the molecules to become more closer to each other thereby resulted to decrease in inter particle spacing which make the volume of the fluid to decrease while density increase. But slight increment in temperature of fluid was observed towards the upper plate as shown in Figure 2(b) due high H_r

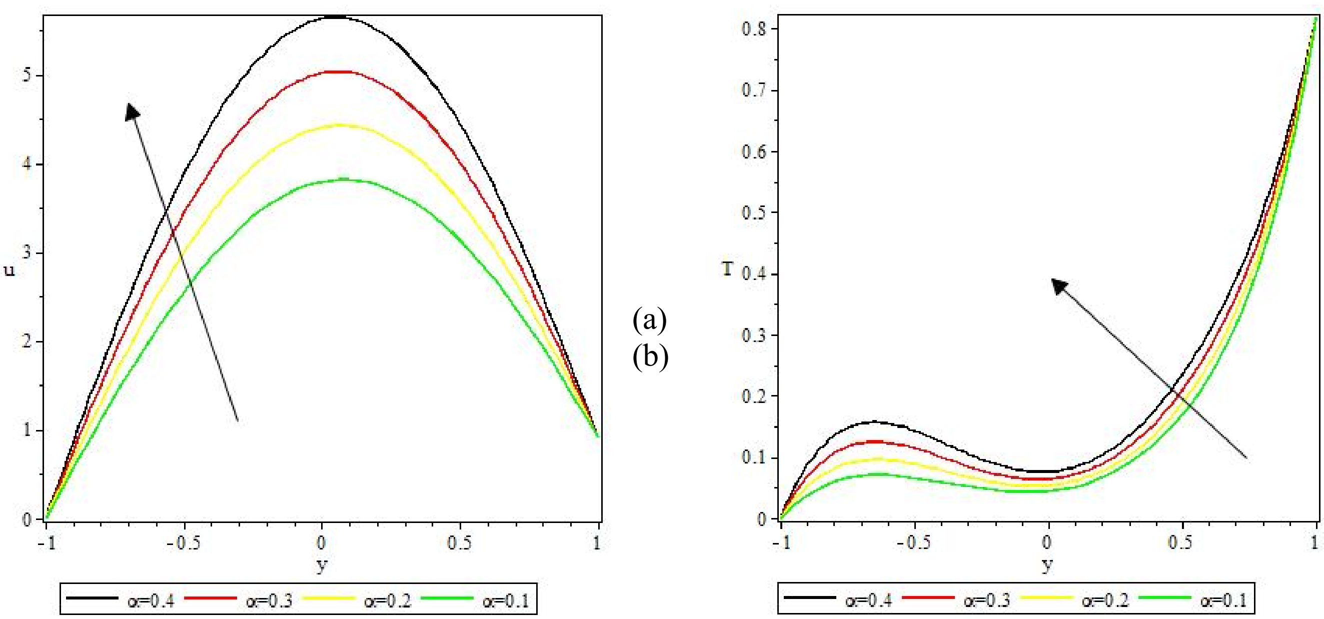


Figure 2: Effects of various values of α on the (a) velocity distribution and (b) temperature distribution at a very high and low H_r when $P_r = 0.7, R_e = 0.5, \eta = 0.5, E_c = 0.01, s = 0.1$ and $\chi = 1.0$

Figure 3 and 4 shows the effect of stratification factor parameter s on the velocity and temperature of the fluid. It was discovered that as s rises, the velocity and temperature of the fluid reduces at a very high and low H_r with reduction in temperature more pronounce at the upper plate. The influence of Reynolds number (Re) on the velocity distribution at a very high and low H_r is illustrated on Figure 5. It is noticed in Figure 5(a) that the velocity distribution reduces with increase in Re at a low H_r while it rises at lower plate and reduces again towards upper plate when H_r is very high as shown in Figure 5(b). This is due to the fact that an increase in Reynolds number leads to a reduction in the viscous forces which in turn oppose the fluid motion at a low H_r . But at high modified magnetic field H_r , the presence of Lorentz force opposes the direction of the flow. This Lorentz force is more dominant than inertial force cause by Reynolds number thereby decrease the velocity of the fluid towards the upper plate.

Bima Journal of Science and Technology, Vol. 8(2B) July, 2024 ISSN: 2536-6041

DOI: 10.56892/bima.v8i2B.704

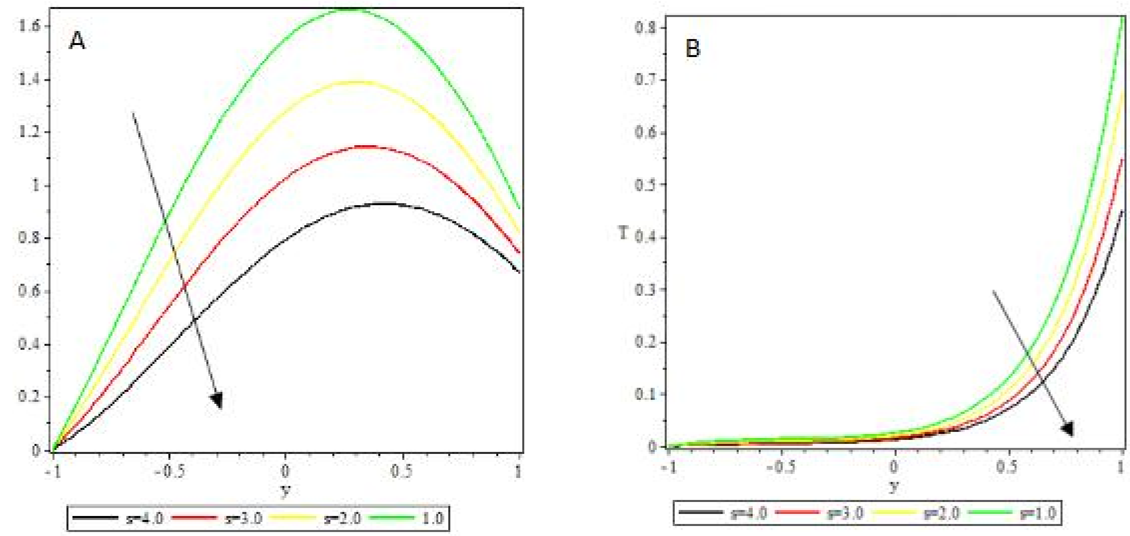


Figure 3: Effects of various values of s on the (a) velocity and (b) temperature distributions at a very high $H_r = 5$ when $P_r = 0.7, R_e = 0.5, \eta = 0.5, E_c = 0.01$ and $\chi = 1.0$

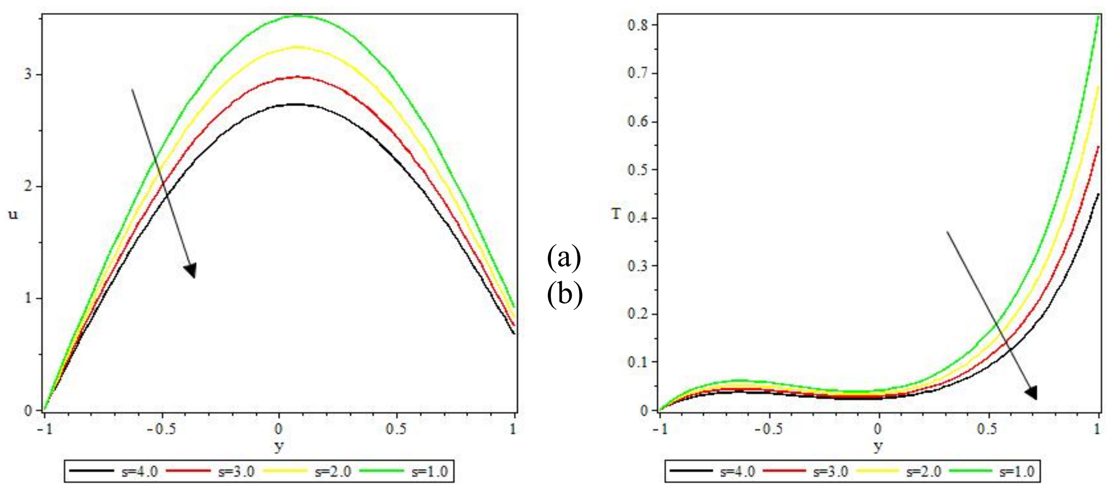
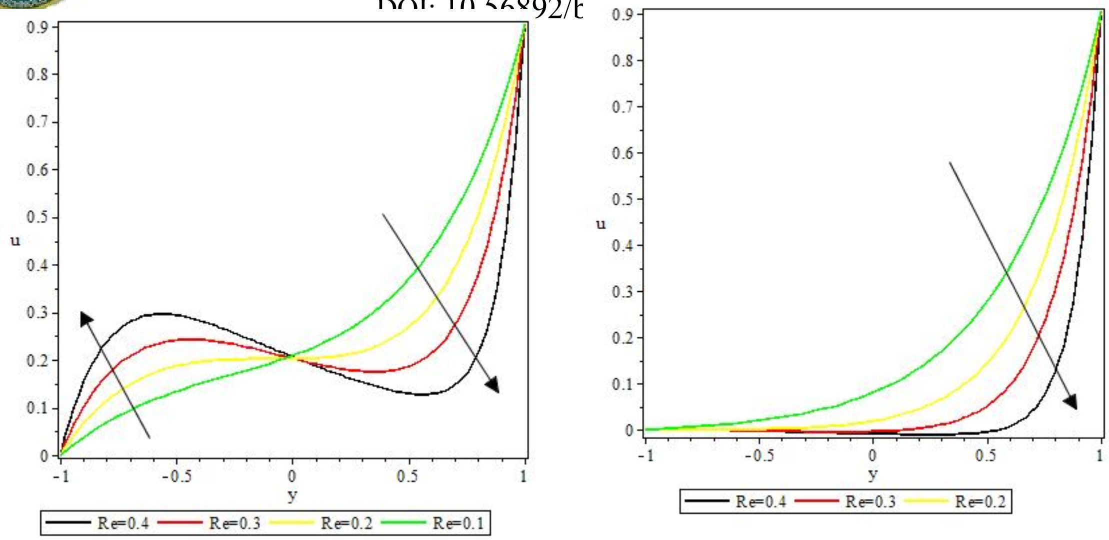


Figure 4: Effects of various values of s on the (a) velocity and (b) temperature distributions at a very low $H_r = 0.2$ when $P_r = 0.7, R_e = 0.5, \eta = 0.5, E_c = 0.01$ and $\chi = 1.0$

The same effects of Re on velocity distribution can be observed for viscoelastic parameter (η) in Figure 6 except that there is slight increment in velocity distribution at lower plate when H_r is very high as shown in Figure 6(b). This occurs as result of Re of shear flow being reduced by η. Figure 7 shows reduction in temperature distribution as Prandtl (Pr) number increases when H_r is high or low. As Pr increases, thermal diffusivity of the fluid diminishes which results in the expansion of the fluid. Because of the presence of H_r the movement of the molecules decreases due to the oppose Lorentz force hence leads to reduction in temperature of the fluid.

DOI: 10.56892/b



(a)
(b)

Figure 5: Effects of various values of Re on the velocity distribution at a very high $H_r = 5.0$ and low $H_r = 0.2$ when $P_r = 0.7, s = 1, \eta = 0.5, E_c = 0.01$ and $\chi = 1.0$

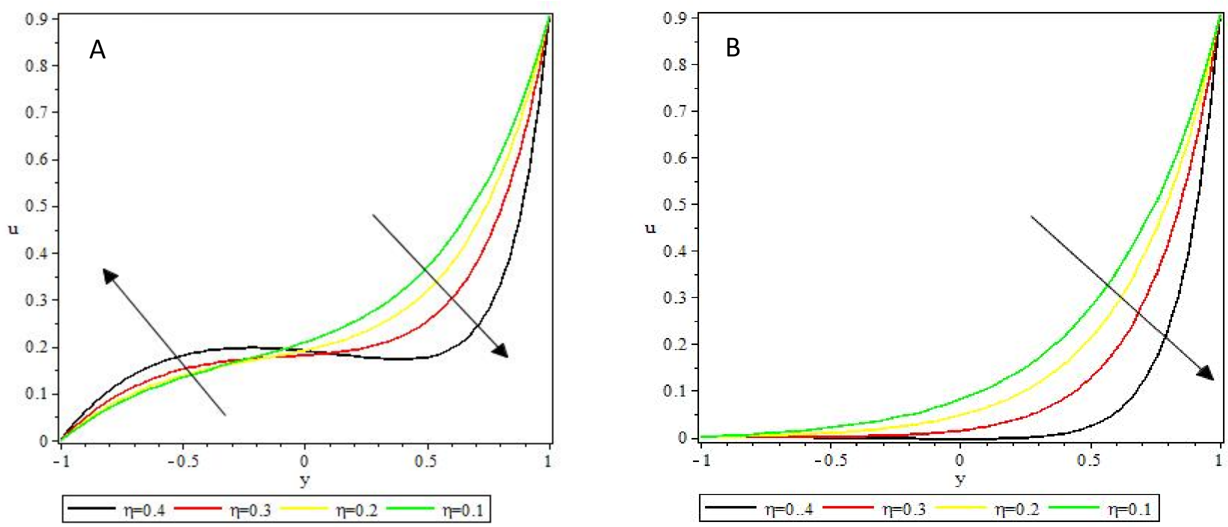


Figure 6: Effects of various values of η on the velocity distributions at a very high $H_r = 5.0$ and low $H_r = 0.2$ when $P_r = 0.7, s = 1, \mathrm{Re} = 0.5, E_c = 0.01$ and $\chi = 1.0$

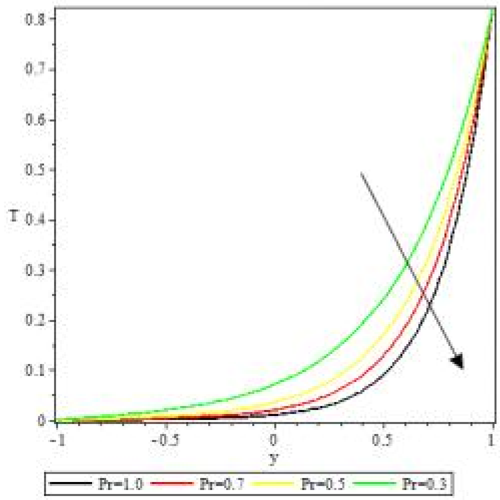


Figure 7: Effects of various values of Pr on the temperature distributions at a very high $H_r = 5.0$ and low $H_r = 0.2$ when $P_r = 0.7, s = 1, \mathrm{Re} = 0.5, E_c = 0.01,$ $\eta = 0.5$ and $\chi = 1.0$

Table 1: Validation of analytical and numerical results for the viscous drag and the rate of heat transfer in term of Nusselt number at the lower plate when H_r is low.

	Analytical results		Numerical results	
Re	τ	Nu	τ	Nu
0.1	0.3848088310	0.5138467550	0.3848090927	0.5138468081
0.2	0.4786293090	0.5074763500	0.4786293100	0.5074763517
0.3	0.5724497870	0.5054417452	0.5724497883	0.5054417458
0.4	0.6662702646	0.5044910895	0.6662702667	0.5044910896

Table 2: Validation of analytical and numerical results for the viscous drag and the rate of heat transfer in term of Nusselt number at the upper plate when H_r is low.

	Analytical results		Numerical results	
Re	τ	Nu	τ	Nu
0.1	0.7497685924	0.4771606994	0.74976865265	0.4771606522
0.2	0.7085488314	0.4895213162	0.70854883238	0.4895213166
0.3	0.6673290710	0.4935946682	0.66732907195	0.4935946689
0.4	0.6261093102	0.4955962054	0.62610931134	0.4955962055

Table 3: Validation of analytical and numerical results for the viscous drag and the rate of heat transfer in term of Nusselt number at the lower plate when H_r is high.

	Analytical results		Numerical results	
Re	τ	Nu	τ	Nu
0.1	0.9273000598	0.5263136060	0.9273000622	0.5263136089
0.2	1.5636117652	0.5269883192	1.5636117707	0.5269883215
0.3	2.1999234741	0.5319988295	2.1999234763	0.5319988319
0.4	2.8362351784	0.5380932882	2.8362351844	0.5380932930

Table 4: Validation of analytical and numerical results for the viscous drag and the rate of heat transfer in term of Nusselt number at the upper plate when H_r is high.

Re	Analytical results		Numerical results	
	τ	Nu	τ	Nu
0.1	0.3520988664	0.4874962282	0.3520988680	0.4874962299
0.2	-0.08679062082	0.4946331332	-0.0867906176	0.4946331332
0.3	-0.5256801070	0.4934827719	-0.5256801055	0.4934827733
0.4	-0.9645695940	0.4902605945	-0.9645695914	0.4902605982

Finally, Table 1 – 4 showed the accuracy and validity of the results in this research and compared it with those obtained numerically at the steady state. These tables shows strong agreement between the two results thereby exhibit the validity of the research methodology.

Table 1 and 2 reveals that the viscous drag force increases at lower plate and slightly decreases at upper plate when H_r is very low. The same things was observed when H_r is very high (i.e Table 3 and 4). This is because the Lorentz force generated by H_r reduced the velocity of the fluid towards the upper plate with high temperature compared to lower plate. For heat transfer (N_u), it reduces ant the lower plate and increases toward the upper plate when H_r is low. But when the H_r is large, it increases at both lower and upper plates.

CONCLUSION

The unsteady movement of viscoelastic stratified fluid between two inclined corresponding Riga plates with viscous dissipation was studied numerically. From the result, it was discovered that modified magnetic field H_r have effect on the flow of the fluid. This effect of H_r was more dominant than the effect of other physical parameter such as pressure, Reynolds number, viscoelastic parameter, Prandtl number and stratification parameter. It is expected that rise in all these parameter will increase the velocity and temperature distribution but this was subdued by the existence of a transverse magnetic field which created Lorentz force and then acted against the path of the flow thereby causing the temperature and the velocity distribution to be decreasing towards the upper plate.

REFERENCES

Gailitis, A. and Lielausis, O. (1961). On a possibility to reduce the hydrodynamical resistance of a plate in an electrolyte. Appl. Magnetohydrodyn, 12: 143-146.

Prakash, O. M., Kumar, D and Dwivedi, Y. K.(2012). Heat transfer in MHD flow of dusty viscoelastic (Walters' liquid model-B) stratified fluid in porous medium under variable viscosity. P RAMANA – Journal of Physics, 49(6), 1457–1470.

Jha, B. K., Samaila, A. K., andAjibade, A. O. (2015). Unsteady natural convection couette flow of a reactive viscous fluid in a vertical channel. Computational Mathematics and Modeling, 24(3):432–442.

Dash, G. C. and Ojha, K. L. (2018). Viscoelastic hydromagnetic flow between two porous parallel plates in the presence of sinusoidal pressure gradient. Alexandria Engineering Journal. 57, 3463-3471.

Sattar J. A. and Abeer, A. J. (2019). A New Analytical-Approximate Solution for the Viscoelastic Squeezing Flow Between Two Parallel Plates. Appl. Math. Inf. Sci.

DOI: 10.56892/bima.v8i2B.704

13(2), 173-182.

Mollah, M. T (2019). EMHD laminar flow of Bingham fluid between two parallel Riga plates. International Journal of Heat and Technology. 37 (2), 641-648

Lawal O. W. and Erinle L. M. (2019). An investigative study of the MHD flow of a third grade fluid in a porous channel under the influence of an induced magnetic field with viscous dissipation. Islamic University Multidisciplanary Journal. 6(3), 187-201

Krishna, M. V.,AmeerAhamad, N. and Chamkha, A. J.(2020). Hall and ion slip effects on unsteady MHD free convective rotating flow through a saturated porous medium over an exponential accelerated plate. Alexandria Engineering Journal. 59, 565–577.

Islam, M. R. and Nasrin, S. (2021). Unsteady couette flow of dusty fluid past between two Riga plates. European Journal of Scientific Research. 159 (2), 18 – 32.

Nasrin S., Mondal, R. N. and Alamet, M. M. (2021). Unsteady couette flow past between two horizontal Riga plates with Hall and ion slip effects. Mathematics and Statistics 9(4), 552-565

Elshabrawy, M., Khaled, O., Abbas, W., Beshir, S and Mostafa A.(2023). Analytical solution of thermal effect on unsteady visco-elastic dusty fluid between two parallel plates in the presence of different pressure gradients. Beni-Suef Univ J Basic App Sci 12:76, 1-14