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Comparative Study of Some Selected Linear and Non-Linear Time Series Models of Different Orders

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ABSTRACT

This study aims to evaluate and contrast the effectiveness of Autoregressive models with modified iterations of Inverse Smooth Transition Autoregressive (ISTAR), Exponential Smooth Transition Autoregressive (ESTAR), and Trigonometric Smooth Transition Autoregressive (TSTAR) models. This research examines the impact of varying sample sizes on these models at different orders. A numerical simulation was done to assess the efficiency of linear and nonlinear models over sample sizes ranging from 20 to 250 for first, second, and third orders. The best-fit model at each order is determined using standard selection criteria, such as Mean Absolute Percentage Error (MAPE), Mean Square Error (MSE), and Akaike Information Criteria (AIC). The model that has the lowest value for the chosen criteria is considered the most optimal. The results suggest that AR models have superior performance for the second order, as determined by AIC and MAPE. On the other hand, TSTAR models exhibit better performance than other models for the third order, as indicated by MSE and AIC.

Keywords: Smooth Transition Autoregressive, functions, Linear and Non-Linear Models.

INTRODUCTION

Time series models are utilised to predict future events by analysing past events that have been observed and collected at regular time intervals. A time series model typically includes the mean component and the conditional variance component. (Box and pierce 1970). A time series is a sequence taken at successive equally space point in time. In fact, time series data consist of an array of both times and numbers. A sequence of clearly defined procedures is followed to implement autoregressive, moving average, exponential, seasonal, and autoregressive moving average modelling. The initial stage involves the identification of the model. The process of identification involves delineating the suitable structure. (Autoregressive (AR) models, Smooth Transition Autoregressive, Exponential Smooth Autoregressive model and Trigonometric model) whether the model is stationary or non-stationary under distributions. Identification is performed by examining plots of the upward trend in time series model, identification is occasionally performed by an automated iterative process that involves fitting numerous potential model structures and ordering is then used to determine the most suitable model.

Autoregressive Mathematical models such as smooth transition and moving average models capture the persistence, or autocorrelation, in a time series. The models are commonly utilised in econometrics, hydrology, engineering, and various other fields. There are various reasons for using Autoregressive (AR) models,

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Smooth Transition Autoregressive (STAR), Exponential Smooth Autoregressive model, and Trigonometric models to analyse data. Modelling can provide valuable insights into the physical system by shedding light on the underlying processes that contribute to the persistence observed in the series. These models can also be utilised to forecast the behaviour of a time series or econometric data based on historical values. This prediction can serve as a reference point for assessing the potential significance of other variables in the system. They are commonly employed to forecast economic and industrial time series. Autoregressive (AR) models, Smooth Transition Autoregressive (STAR), Exponential Smooth Autoregressive model, and Trigonometric model can also be used for simulation purposes. These models allow for the generation of synthetic series that mimic the persistence structure of an observed series. Simulations are particularly valuable in establishing confidence intervals for statistics and estimated econometrics quantities.

Autoregressive, autoregressive moving average, moving average, exponential, and seasonal modelling follow a clear set of steps. Identifying the model is the initial step. Identification involves determining the suitable structure for the model, whether it is stationary or non-stationary under different distributions. This can be done using various models such as Autoregressive (AR) models, Smooth Transition Autoregressive models, Exponential Smooth Autoregressive models, and Trigonometric models. Identification can be performed by examining plots of the autocorrelation function and partial autocorrelation function. Identification is often achieved through an automated iterative process that involves fitting various model structures and orders. A goodness-of-fit statistic is then used to determine the most suitable model.

LITERATURE REVIEW

A time series is a sequence of data points, measured typically at successive points in time spaced at uniform time intervals. Examples of time series are the daily closing value of the Dow Jones Industrial Average and the annual flow volume of the Nile River at Aswan (Horváth, L., Miller, C., and Rice, G. (2020)). Time series are very frequently plotted via line charts. Time series are used in statistics, signal processing, pattern recognition, econometrics, mathematical finance, weather forecasting, earthquake prediction, electroencephalography, control engineering, astronomy, and communications engineering . (Brockwell, P.J. and Davis, R.A. (2002))

Another definition of a time series is that of a collection of quantitative observations that are evenly spaced in time and measured successively. Examples of time series include the continuous monitoring of a person's heart rate, hourly readings of air temperature, daily closing price of a company stock, monthly rainfall data, and yearly sales figures. Time series analysis is generally used when there are 50 or more data points in a series. If the time series exhibits seasonality, there should be 4 to 5 cycles of observations in order to fit a seasonal model to the data, Brockwell, P.J. and Davis, R.A. (2002).

Models for time series data can have many forms and represent different stochastic processes. When modeling variations in the level of a process, three broad classes of practical importance are the autoregressive (AR) models, the integrated (I) models, and the moving average (MA) models. These three classes depend linearly on previous data points. Combinations of these ideas produce autoregressive moving average (ARMA) and autoregressive integrated moving average (ARIMA) models. The autoregressive fractionally integrated moving average

(ARFIMA) model generalizes the former three, Alok, M. S. (2020).

The autoregressive (AR) model is used for future performances in time series model. When analyzing time series data, it is common to discuss the concept of stationarity, which refers to the behavior of a specific variable over time. There are three components to stationarity. The series exhibits a constant mean, indicating that there is no inherent inclination for the mean of the series to fluctuate over time. Additionally, it is assumed that the variance of the series remains constant over time. It is generally believed that the autocorrelation pattern remains consistent throughout the series. Over the past twenty years, numerous non-linear time series models have been put forward. These include the random coefficient of AR model by (Ratnasingam,S.,and Ning,W (2020)), the amplitude dependent exponential AR (EX PAR) model by Maulana, A., and Slamet, I. (2020), the threshold AR model by Fan, J.Q. (2019), and the bilinear model by Granger and Andersen (1978), among several others.

These studies suggest that, in trying to decide by classical methods whether economic data are stationary or integrated, it would be useful to perform tests of the null hypothesis of stationarity as well as tests of the null hypothesis of a unit root. This paper provides a straight forward test of the null hypothesis of stationarity against the alternative of a unit root at different order of autoregressive and moving average and various sample sizes. There have been surprisingly few previous attempts to test the null hypothesis of stationarity. Park and Mahdi,E. (2020) consider a test statistic which is essentially the F statistic for 'superfluous' deterministic trend variables; this statistic should be close to zero under the stationary null but not under the alternative of a unit root. Zhang,J and Zhou,B (2022) considers the Dickey-Fuller test statistics, but estimates both trend-stationary and difference-stationary models and then uses the bootstrap to evaluate the distribution of these statistics.

MATERIALS AND METHODS

Simulation studies was conducted to investigate the performances of Tests of stationarity under different orders of Autoregressive with Modified version models.

$Y_t = \emptyset_1 Y_{t-1} + \emptyset_2 Y_{t-2} + \ldots + \emptyset_p Y_{t-p} + e_t$, --------------------- 3.0 where e_t is white noise process with zero mean and variance σ^2. $\emptyset_1, \emptyset_2, \ldots \emptyset_p$ are autoregressive parameters. The mathematical model for existing and proposed models are

ISTAR$_{(p)}$ is $Y_t = \frac{1}{\phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \ldots + \phi_p Y_{t-p} + e_t}$ -------------------- 3.1 is called Inverse Smooth Transition Autoregressive model. Where e_t is white noise process with zero mean and variance(σ^2), Y_t is variable of interest at time (t), $\phi_1, \phi_2, \ldots \phi_p$ are the coefficient that define the unit root.

ESTAR$_{(p)}$ is $Y_t = e^{\phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \ldots + \phi_p Y_{t-p} + e_t}$ ---------------------- 3.2 is called Exponential Smooth Transition Autoregressive model.

TSTAR$_{(p)}$ is $Y_t = Sin\phi_1 Y_{t-1} + Sin\phi_2 Y_{t-2} + \ldots + Sin\phi_p Y_{t-p} + e_t$ --------------------- 3.3 is called Trigonometric Smooth Transition Autoregressive model.

Selection rule

Mean Absolute Percentage Error (MAPE), Mean Square Error (MSE), and Akaike Information Criteria (AIC) were calculated for the given sample sizes. (n)=20, 50, 70, 100, 120, 150, 180, 200, 230 and 250 for each model, determine the model with the lowest values for these criteria, which were considered to be the optimal model. In this study order 1,2 and 3 of each models are considered in respective case and situation.

Results, presented in tables 1 to 3, show that AR models are the best fit for the first order based on MAPE and MSE, AR models for the second order based on AIC and MAPE,and TSTAR models for the third order based on MSE and AIC.

RESULTS AND DISCUSSION

Simulation study were conducted for the Autoregressive models and proposed models such as ISTAR, ESTAR and TSTAR at different order under stationary and to assess and compare the performances, and to investigate the Effect of sample size of each models at different orders were examined on each of the models data simulated

Results, presented in tables 1 to 3, show that AR models are the best fit for the first order based on MAPE and MSE, AR models for the second order based on AIC and MAPE,and TSTAR models for the third order based on MSE and AIC.

For the simulation study, the parameters of the linear and nonlinear Autoregressive functions in models 1-4 are fixed as $\phi_1 = 0.5, \phi_2 = 0.3\ and\ \phi_3 = 0.1$. The sample sizes simulated from each of model cases are; $20,50,70,100,120,150, 180, 200, 230\ and\ 250$. The simulation study was conducted 1000 times for various versions of the model functions, using a certain sample size.

Table 1: Performances of the fitted models for the first order on the Basis of MAPE and MSE criteria.

Sample	MAPE				MSE			
Size(n)	AR	ISTAR	ESTAR	TSTAR	AR	ISTAR	ESTAR	TSTAR
20	10.0674	11.1344	12.159	11.1802	11.142	11.871	12.358	11.3786
50	10.0647	11.1272	12.1383	11.1409	11.059	11.287	12.311	11.1587
70	10.0421	11.0122	11.0484	11.0981	11.042	11.168	12.108	11.1492
100	10.964	10.9966	10.9768	11.0980	11.020	11.166	11.0865	11.1454
120	10.9481	10.9905	10.9676	11.0707	10.935	11.139	11.0642	11.141
150	10.9018	10.9764	10.9863	10.065	10.891	11.096	11.0553	11.0710
180	10.8956	10.9661	10.9147	10.9639	10.888	11.051	11.9663	11.0464
200	10.8767	10.9428	10.9208	10.946	10.887	10.947	11.9421	11.0459
230	10.8748	10.9142	10.8939	10.9079	10.866	10.907	11.9208	11.9079
250	10.8716	10.8344	10.8915	10.8878	10.731	10.839	10.9147	11.8878

Table 2: Performances of the fitted models for the second order on the Basis of AIC and MAPE criteria

Sample	AIC				MAPE			
Size(n)	AR	ISTAR	ESTAR	TSTAR	AR	ISTAR	ESTAR	TSTAR
20	10.064	11.1314	11.156	12.1772	12.0584	11.0254	14.15	11.1712
50	10.061	11.1242	11.1353	12.1379	12.0557	11.0182	14.1293	11.1319
70	10.039	11.0092	11.0454	12.0951	12.0331	11.0032	13.0394	11.0891
100	9.961	10.9936	10.9738	11.077	11.9555	10.9376	12.9678	11.0713
120	9.9451	10.9875	10.9646	11.0677	10.9391	10.9215	12.9586	10.0617
150	9.8988	10.9734	10.9833	11.062	10.8928	10.8674	11.9773	10.056

180	9.8926	10.9631	10.9117	10.9609	10.8866	10.8571	11.9257	10.9549
200	9.8737	10.9398	10.9178	10.943	10.8677	10.8338	11.9118	10.937
230	9.8718	10.9112	10.8909	10.9049	10.8658	10.8052	10.8849	10.8989
250	9.8686	10.8314	10.8691	10.8848	10.8626	10.8254	10.8631	10.8788

Table3: Performances of the fitted models for the third order on the Basis of MSE and AIC criteria

Sample Size(n)	MSE				AIC			
	AR	ISTAR	ESTAR	TSTAR	AR	ISTAR	ESTAR	TSTAR
20	12.1039	11.8321	12.3198	10.0396	10.0264	11.0934	12.118	10.0092
50	12.0206	11.2487	12.2722	10.0197	10.0237	11.0862	12.0973	10.0089
70	11.0033	11.1293	12.069	10.0102	10.0011	10.9712	12.0074	10.0071
100	10.9818	11.1274	11.0475	10.0064	10.923	10.9556	11.9358	9.9069
120	10.8964	11.1001	11.0252	10.0042	10.9071	10.9495	11.9266	9.8965
150	10.8526	11.0577	10.0163	10.032	10.8608	10.9354	11.9453	9.8040
180	10.8499	11.0124	10.9273	10.0074	10.8546	10.9251	11.8737	9.8229
200	10.8485	10.9083	10.9031	10.0069	10.8357	10.9018	11.8708	9.8050
230	10.827	10.8687	10.8818	10.0065	10.8338	10.8732	11.8529	9.8025
250	10.6927	10.8006	10.8757	10.0044	10.8306	10.7934	11.8311	9.8018

CONCLUSION

In this study, the comparison of Autoregressive (AR) models with modified versions of Inverse Smooth Transition Autoregressive (ISTAR), Exponential Smooth Transition Autoregressive (ESTAR), and Trigonometric Smooth Transition Autoregressive (TSTAR) models were done. An extensive simulation study was conducted for each model across sample sizes of 20, 50, 70 100, 120, 150, 180, 200, 230 and 250. Standard criteria, including MAPE, MSE, and AIC, were employed to identify the best model, with the model exhibiting the lowest criteria values considered the best. Results, presented in tables 1 to 3, show that AR models are the best fit for the first order based on MAPE and MSE, AR models for the second order based on AIC and MAPE,and TSTAR models for the third order based on MSE and AIC.

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