



## Flood Routing of River Kaduna and its Catchment Using Muskingum Model

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### ABSTRACT

This paper aims to use Muskingum Models to simulate the storage volume of flooding in the Kaduna River channel and generate the outflow hydrograph by routing the inflow. The simulation was based on thirty (30) years of monthly inflow stream data for the Kaduna River, with the peak value for each year being used. The hydrograph clearly depicts the flow pattern between the inflow and outflow, with the outflow lagging the inflow by around 2.3 hours, which was the value of K utilized in the computations and represents the transit time in the reach. The peak inflow is 801.18 m<sup>3</sup>/s, with an outflow of 736.74 m<sup>3</sup>/s; the storage level at this time is 64.44 cumesc (flood advancement). On the other hand, the highest outflow is 756.99 m<sup>3</sup>/s at an input of 706.30 m<sup>3</sup>/s, with a recessive storage of 50.69 m<sup>3</sup>/s, which is the volume of inundation at that moment (volumetric rate lost by the river). The average storage is a negative differential (-1.061 m<sup>3</sup>/s), while an approximate storage balance was maintained at an inflow and outflow of 694.16 m<sup>3</sup>/s and 694.46 m<sup>3</sup>/s respectively. The average negative storage differential implies that the outflow exceeds the inflow at this point, thereby shrinking the capacity of the river and possibly causing surface inundation downstream, as more outflows exceed the inflows. The model validation shows a good correlation ( $r^2=0.8989$ ) between the inflow and outflow. The Muskingum technique has shown to be an efficient model for flood routing, based on the result, hence the study recommends it for river routing in areas with flood history.

**Keywords:** Flow routing, Muskingum model, Kaduna River, Outflow Hydrograph

### INTRODUCTION

Flood routing is critical to the design of both structural and nonstructural flood control techniques (Barati *et al.*, 2018). Routing is the process of calculating changes in the amplitude, velocity, and form of a flood wave over time at one or more places along a river (Hamed *et al.*, 2016). Flood routing methods are classified into two types: hydraulic and hydrological. Hydraulic methods contain sophisticated computations and are more data-intensive, but represent the whole flood wave profile, whereas hydrological approaches are

considerably easier, but produce the flood hydrograph at the end of a reach (Zhang and Kang, 2017; Dottori and Todini, 2013). Hydrologic approaches require simply the inflow hydrograph for a river stretch. The Muskingum method is a popular hydrologic approach with numerous variations and parameters ranging from two to five. The Muskingum method with two or three parameters is more prevalent (Omran *et al.*, 2024). In recent years, optimization methods, particularly evolutionary algorithms, for



predicting Muskingum parameters have gained popularity (Zhang and Kang, 2017).

Costabile *et al.*, 2017 reported that the main advantages of the one-dimensional model over the 2-D models for flood routing are a simpler run process and low computational time where topographic data was unnecessary. Fassoni-Andrade *et al.*, (2018) considered the development of a one-dimensional model based on the equation of hydrological models, which include the continuity equation and mass equation, such as the equation of the Muskingum models (Fassoni-Andrade *et al.*, 2018). It was observed that one-dimensional flow routing inertial models, based on the explicit solution were superior to the other models. These models simplified the Saint-Venant equation, and the main advantages of these models were good simulated results with a simple structure. Singh and Arvamuthan, (1996) applied two hydraulic models that were developed based on the Kinematic and diffusion waves, in addition, the results were compared to the Muskingum model for the flood routing. The results showed that the simulation of hydraulic models was dependent on the kinematic wave number, so that when the value of this parameter was not considered based on accurate computation, the results for the hydraulic model could be worse than hydrologic models. Costabile *et al.*, (2015) reported that 2-D models could overcome the limitation of 1-D models, when the case study characterized as unsteady flow in irregular topography. The reports showed that if the flow was not one dimensional for the urban hydraulic, the one-dimensional channel network should be used instead of a one-dimensional model. In addition, the results showed the significant difference between 1-D and 2-D models to simulate the velocity and depths.

However, the results showed that the complex nonlinear form characteristic, numerical

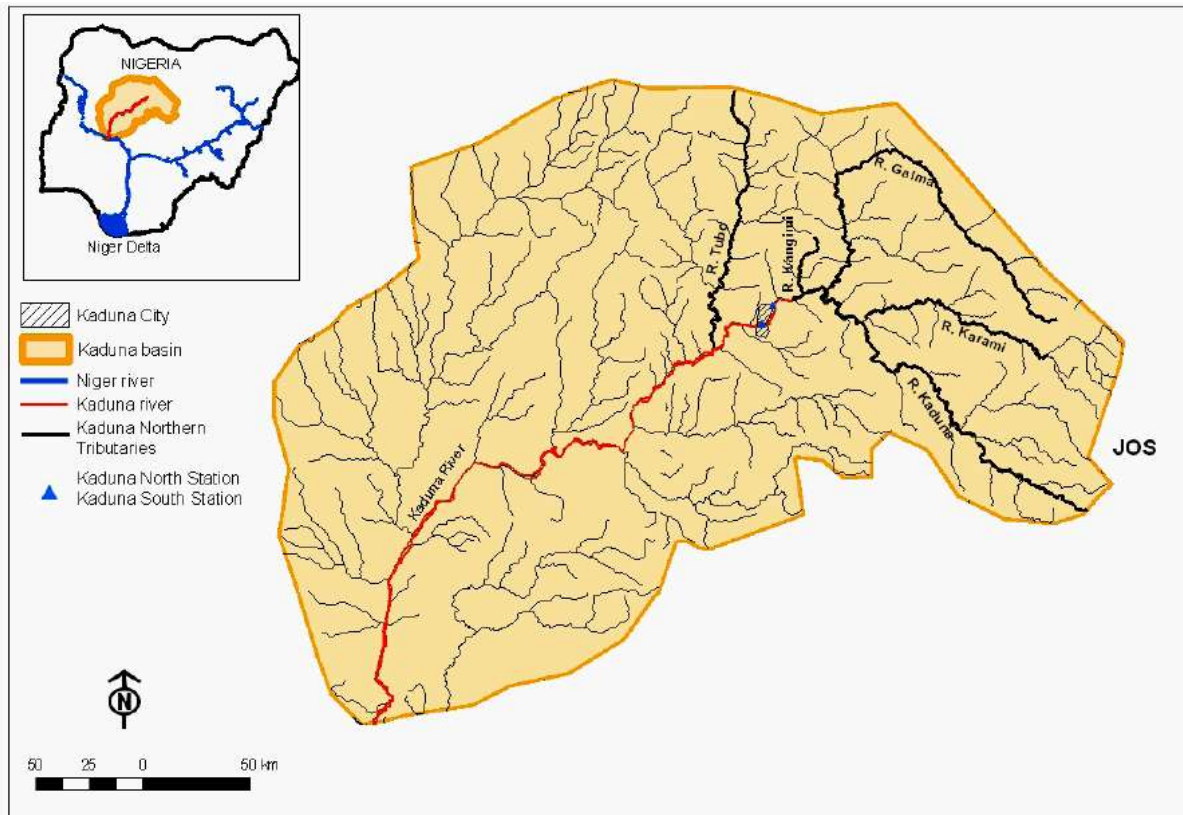
stability, high computational time, and complexity in the run process of hydraulic models, meant that the simpler and more accurate models have high importance (Costabile *et al.*, 2017). In fact, the hydraulic models need to measure the flow depth and discharge based on applying stream gaging. These models are known as complex models and difficult to use, whilst the hydrologic models need only to use the discharge data. In addition, the hydrologic models can be effective for the initial planning level, where the measuring system is undeveloped for accurate measurement (Costabile *et al.*, 2017). For example, Chatila, (2003) simulated flood routing based on the Muskingum model and EXTRAN hydraulic model. The hydraulic model developed was based on finite difference. Both hydrologic models and hydraulic models were applied on simple and compound channels for flood routing. The results revealed that the Muskingum model had achieved higher accuracy compared with the hydraulic model because of its flexibility in calibration, where even the river bed geometry was not considered for this model. It has been demonstrated that the Muskingum model could simulate the peak discharge, achieving a close fit with the actual one, compared to the hydraulic model (Moradi *et al.*, 2023; Kadhim *et al.*, 2022). Furthermore, it has been reported that hydraulic models are dependent too many assumptions, such as reach geometry, channel slope, and flow velocity, which causes the application of some hydraulic models to be limited to the specific case studies.

This paper seeks to model the storage volume of flooding in Kaduna River channel in order to generate the outflow hydrograph by routing the inflow, using Muskingum Models.

## Study location

The Kaduna river took its source on the Jos Plateau, flows northwest across the Kaduna plains cutting several gorges through rugged terrain between Kaduna and Zungeru. Finally, the river flows southwards through the broad, level Niger valley, and enters the Niger River near Wuya in Niger State having drained about 70,200 square kilometers of land area in a 550km long main river course (Arimoro, *et al.*, 2018) covering Kaduna, Niger, FCT, parts

of Plateau, Nasarawa, and Kano States. Major tributaries joining the Kaduna River along its course include rivers Karami, Galma, Tubo, Sarkin Pawa and Mariga in that order from source. Kaduna is the only state capital the main channel passes through and Shiroro hydropower reservoir is the only major dam across the main. Figure 1 presents the map of the Kaduna river basin.



**Figure 1:** Map of the Kaduna River Basin: Source (Alayande, 2010)

## MATERIALS AND METHODS

### Flood Routing Model Description

The Muskingum method of flood routing is based on the continuity equation and a storage-discharge relation (Salvati *et al.*, 2024). Previous study considers the lateral flow for the flood routing based on a ration of

inflow rate  $O_{lateral} = \beta I_t$  while the other studies do not consider the effect of lateral flow for flood regime, and thus it adds one term to the equation of the Muskingum model with three parameters. If  $b$  coefficient equals to zero, the lateral flow has not been considered for the flood routing.

$$\frac{ds_t}{dt} = O_t - (1 + \beta)I_t \quad 1$$

$$S_t = K[X(1 + \beta)I_t + (1 - X)O_t] \quad 2$$

where  $O_t$  is the output flow at time  $t$ ,  $I_t$  is the input flow at time  $t$ ,  $S_t$  is the storage at time  $t$ ,  $\frac{ds_t}{dt}$  is the storage time variation at time  $t$ ,  $K$  is the time of travel of the flood wave through the channel reach and  $X$  is the weighting factor showing the effect of input and output flows on storage.

Equation (2) expresses a linear relation between storage, input and output flows. However, a nonlinear relationship between the three parameters (storage, input and output flows) is also presented as:

$$S_t = K[X(1 + \beta)I_t + (1 - X)O_t]^m \quad 3$$

Where  $m$  is an exponent using equations 1 and 3; one can obtain (Barati, 2011)

$$O_t = \frac{1}{1-X} \left(\frac{S_t}{K}\right)^{\frac{1}{m}} - \left(\frac{X(1+\beta)I_t}{1-X}\right) \quad 4$$

To ensure the accuracy of the Muskingum model,  $C_1 + C_2 + C_3 = 1$  11

$Q_j$  = The outflow at a given time,  $t$

$Q_{t+1}$  = The next outflow at time  $t+1$

$I_j$  = The inflow at time,  $t$

$I_{j+1}$  = The next inflow at time,  $t+1$

The first value of outflow,  $Q_1$  is assumed using the statistical principle of assume mean, while the next  $j+1$  outflow is computed from equation 7

### Data Source

Thirty (30) years Inflow Stream data (1979-2008) for Kaduna River was obtained from the Kaduna South Waterworks gauging station as presented in Table 1. The flow measurements are in cubic metre per second ( $m^3/s$ ).

$$\frac{\Delta S_t}{\Delta t} = -\left(\frac{1}{1-X}\right)\left(\frac{S_t}{K}\right)^{\frac{1}{m}} + \left(\frac{X(1+\beta)I_t}{1-X}\right) \quad 5$$

Using  $S_t$ ,  $\Delta S_t$  the storage later can be expressed as:

$$S_{t+1} = S_t + \Delta S \quad 6$$

The computation of parameter for this work is based on linear model expressed below:

The outflow at a time  $j+1$  is expressed as;

$$Q_{t+1} = C_1 I_{j+1} + C_2 I_j + C_3 Q_j \quad 7$$

Where:

$$C_1 = \frac{\Delta t - 2KX}{2K(1-X) + \Delta t} \quad 8$$

$$C_2 = \frac{\Delta t + 2KX}{2K(1-X) + \Delta t} \quad 9$$

$$C_3 = \frac{2K(1-X) - \Delta t}{2K(1-X) + \Delta t} \quad 10$$

## RESULTS AND DISCUSSION

Equations 7-11 are used to analyze the results and compose the model. The  $K$  value, which represents the time of transit of the flood wave across the channel reach, is considered to be 2.3 hours, with a routing period of one hour intervals. For the linear reservoir model used in this study, the weighting factor  $X$  varies from 0 to 0.3. Table 2 shows the model development for flow routing across the Kaduna River reach using the Muskingum approach. The initial outflow ( $Q_1$ ) is considered to be 721 cubic metre per second ( $m^3/s$ ), with a weighting factor ( $X$ ) of 0.15.



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**Table 1:** Monthly inflow data for Kaduna River in cubic metere per second (m<sup>3</sup>/s)

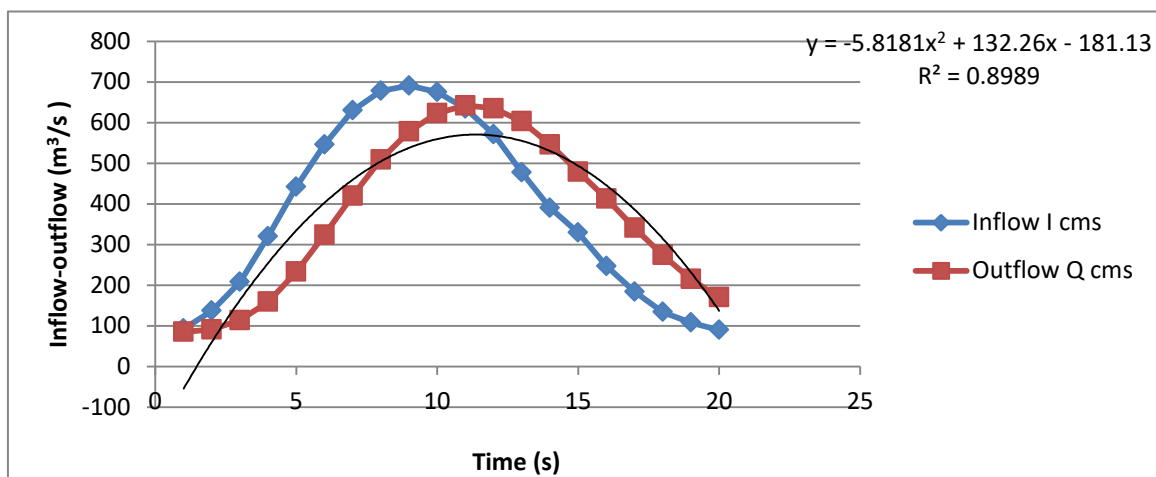
year	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	Peak flow(m <sup>3</sup> /s)
1979	64.78	61.50	41.46	55.63	66.35	123.41	325.05	731.79	703.00	382.44	189.31	97.02	731.79
1980	62.70	56.23	57.17	59.93	77.41	150.83	351.30	769.15	661.13	370.86	185.68	99.29	769.15
1981	64.29	60.85	53.41	53.39	77.67	160.10	273.36	684.06	737.11	418.72	183.11	105.20	737.11
1982	65.49	56.47	40.71	38.05	57.67	120.38	302.33	699.86	751.33	429.20	193.70	101.41	751.33
1983	57.79	47.73	52.00	46.03	63.37	108.89	287.53	635.78	713.26	409.60	199.73	105.99	713.26
1984	56.80	50.14	49.20	47.42	71.01	117.91	324.09	716.49	701.50	354.94	178.48	101.16	716.49
1985	56.24	49.54	41.43	50.58	57.59	138.36	317.96	760.41	763.63	400.25	176.26	107.05	763.63
1986	60.71	58.90	51.24	48.58	69.97	116.09	283.01	690.31	636.24	375.61	175.78	89.22	690.31
1987	55.43	51.70	44.43	47.08	70.93	140.43	327.91	774.75	748.29	412.91	202.95	105.93	774.75
1988	64.52	61.11	48.13	53.79	60.60	120.26	270.08	664.35	610.80	363.04	181.35	93.23	664.35
1989	57.80	49.59	48.75	45.59	72.72	136.96	323.54	783.31	786.46	391.42	190.29	94.41	786.46
1990	64.09	56.83	49.01	42.45	72.89	151.41	333.89	648.61	660.05	394.40	187.88	87.95	660.05
1991	59.98	54.87	46.76	59.95	64.23	139.58	299.37	674.28	631.93	335.59	185.02	93.45	674.28
1992	65.56	53.34	44.09	38.70	62.67	143.42	285.69	651.04	694.16	394.06	195.14	108.81	694.16
1993	57.86	47.88	41.55	56.21	69.56	116.92	289.57	652.62	642.23	340.86	156.87	91.82	652.62
1994	56.78	53.40	40.98	38.19	67.83	136.66	318.36	707.35	744.32	393.14	200.81	87.89	744.32
1995	59.81	54.86	43.97	38.43	61.69	133.08	260.91	700.82	751.95	408.35	189.93	101.21	751.95
1996	61.27	50.69	53.99	58.40	63.11	115.30	296.39	650.08	624.03	394.63	181.88	88.51	650.08
1997	59.69	53.57	39.96	55.66	66.34	121.97	284.83	617.86	657.07	375.58	168.15	92.61	657.07
1998	55.55	53.67	55.89	58.45	68.99	145.77	299.66	772.63	681.90	370.99	195.52	107.31	772.63
1999	59.83	50.98	44.55	37.94	56.66	125.84	302.05	730.20	637.58	368.32	169.57	92.43	730.20
2000	65.90	52.70	44.85	40.65	62.25	120.01	282.71	693.08	625.52	345.85	179.49	87.87	693.08
2001	59.82	51.39	53.94	57.66	68.06	122.36	284.25	648.16	645.12	388.04	197.91	92.80	648.16
2002	63.06	58.31	40.56	36.85	57.96	114.79	277.82	762.98	758.30	427.34	212.82	110.69	762.98
2003	66.50	59.40	47.66	56.88	76.53	126.92	305.97	725.66	669.37	377.98	171.55	90.07	725.66
2004	63.43	53.29	48.48	41.86	59.21	140.50	302.60	788.35	780.46	441.63	189.92	90.75	788.35
2005	63.08	57.34	40.14	52.12	68.06	127.22	316.44	709.55	709.53	391.16	195.94	111.30	709.55
2006	57.74	52.17	50.10	46.77	73.83	155.98	329.30	801.18	661.83	377.75	173.91	88.84	801.18
2007	57.45	46.14	45.67	58.22	77.86	125.47	247.86	649.48	706.30	421.21	180.61	109.84	706.30
2008	65.81	55.56	42.02	50.34	76.96	141.32	307.67	659.95	626.59	380.81	176.21	94.43	659.95

Source: Kaduna South Waterworks gauging station

**Table 2:** Routed flood model through Kaduna River by Muskingum

Routing Period (h)	Inflow I (m <sup>3</sup> /s)	c1Ij+1(m <sup>3</sup> /s)	C2Ij (m <sup>3</sup> /s)	C3Qj(m <sup>3</sup> /s)	Q (m <sup>3</sup> /s)	C1	C2	C3
						0.0631	0.3442	0.5927
1	731.79				721			
2	769.15	48.533365	251.8821	427.3367	727.7522	<b>C1+C2+C3 =1.0</b>		
3	737.11	46.511641	264.7414	431.3387	742.5918			
4	751.33	47.408923	253.7133	440.1342	741.2563			
5	713.26	45.006706	258.6078	439.3426	742.9571			
6	716.49	45.210519	245.5041	440.3507	731.0653			
7	763.63	48.185053	246.6159	433.3024	728.1033			
8	690.31	43.558561	262.8414	431.5468	737.9468			
9	774.75	48.886725	237.6047	437.3811	723.8725			
10	664.35	41.920485	266.669	429.0392	737.6287			
11	786.46	49.625626	228.6693	437.1925	715.4874			
12	660.05	41.649155	270.6995	424.0694	736.4181			
13	674.28	42.547068	227.1892	436.475	706.2113			
14	694.16	43.801496	232.0872	418.5714	694.4601			
15	652.62	41.180322	238.9299	411.6065	691.7167			
16	744.32	46.966592	224.6318	409.9805	681.5789			
17	751.95	47.448045	256.1949	403.9718	707.6148			
18	650.08	41.020048	258.8212	419.4033	719.2445			
19	657.07	41.461117	223.7575	426.2962	691.5149			
20	772.63	48.752953	226.1635	409.8609	684.7773			
21	730.2	46.07562	265.9392	405.8675	717.8824			
22	693.08	43.733348	251.3348	425.4889	720.5571			
23	648.16	40.898896	238.5581	427.0742	706.5312			
24	762.98	48.144038	223.0967	418.761	690.0018			
25	725.66	45.789146	262.6177	408.964	717.3709			
26	788.35	49.744885	249.7722	425.1857	724.7028			
27	709.55	44.772605	271.3501	429.5313	745.654			
28	801.18	50.554458	244.2271	441.9491	736.7307			
29	706.3	44.56753	275.7662	436.6603	756.994			
30	659.95	41.642845	243.1085	448.6703	733.4216			

Source: Author’s Simulation, 2024



**Figure 2:** Inflow-Outflow hydrograph of Kaduna River

Figure 2 depicts the inflow and outflow hydrographs plotted against time, revealing a clear flow pattern between the two. It is clear that the outflow trails the intake by around 2.3 hours, which was the value of  $K$  utilized in the calculations and represents the travel time in the reach.

The average inflow and outflow as obtained from the routed model in Table 2 are  $719.373 \text{ m}^3/\text{s}$  and  $720.43 \text{ m}^3/\text{s}$  respectively. The storage, which is the difference between inflow and outflow, shows flood advancement and recession for positive and negative differentials, respectively. The average storage differential is  $-1.061 \text{ m}^3/\text{s}$ , but an approximate balance of storage was maintained with inflow and outflow of  $694.16$  and  $694.46 \text{ m}^3/\text{s}$ , respectively. The average negative storage differential ( $-1.061 \text{ m}^3/\text{s}$ ) indicates that the outflow is more than the inflow at this location, thereby reducing the river's capacity and possibly triggering surface inundation. The peak inflow is  $801.18 \text{ m}^3/\text{s}$ , with an outflow of  $736.74 \text{ m}^3/\text{s}$ ; storage at this time is  $64.44 \text{ m}^3/\text{s}$  (flood advance). On the other hand, the highest outflow is  $756.99 \text{ m}^3/\text{s}$  with a corresponding inflow of  $706.30 \text{ m}^3/\text{s}$ , with a recessive storage of  $50.69 \text{ m}^3/\text{s}$ , which represents the volume of inundation at that moment (volumetric rate lost by the river). To validate the model, a trendline was utilized to create a model for the inflow-outflow relationship. Based on this, the model created has a coefficient of determination of ( $r^2 = 0.8989$ , and the correlation coefficient is the square root ( $r = 0.95$ ). The correlation value (0.95) indicates a good correlation between input and outflow, which validates the model. In addition, the sum of Muskingum constants  $C1$ ,  $C2$ , and  $C3$  yields a unity (1.0), which is another validation method, with a very high correlation value (1.0) between the inflow and outflow.

## CONCLUSION

The Muskingum model was used to route monthly stream flow data from the Kaduna River over a thirty-year period. Given the provided input information, the model could construct the system's outflow hydrograph. The inflow-outflow hydrograph shows that the outflow lags the inflow. The average storage differential is negative ( $-1.061 \text{ m}^3/\text{s}$ ), with an approximate storage balance of  $694.16 \text{ m}^3/\text{s}$  inflow and  $694.46 \text{ m}^3/\text{s}$  outflow. The average negative storage differential suggests that the outflow exceeds the inflow at this location, hence diminishing the river's capacity and perhaps producing surface flooding downstream, as additional outflows exceed the inflows. The Muskingum technique is an efficient model for flood routing, as model validation demonstrates a strong correlation between inflow and outflow, despite the fact that some case studies have observed numerical instability with the employed numerical methods. The Muskingum technique has shown to be an efficient model for flood routing, based on the result, hence the study recommends it for river routing in areas with flood history.

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