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BAYESIAN ESTIMATION OF THE SHAPE PARAMETER OF GENERALIZED INVERSE EXPONENTIAL DISTRIBUTION UNDER NON-INFORMATIVE PRIORS

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ABSTRACT

In this paper, the shape parameter of the Generalized Inverse Exponential Distribution (GIED) was estimated using both maximum likelihood and Bayesian estimation techniques. The Bayes estimates are obtained using squared error loss function and precautionary loss function by considering Uniform prior, Jeffery's prior and Extended Jeffery's prior. Thus the study considered these three priors under the square error loss function and also the same three priors under precautionary loss function to derive the estimators for the shape parameter. The estimator with minimum posterior risk and mean square error (MSE) as criteria is selected as the best. To achieve this, an extensive Markov Chain Monte Carlo (MCMC) simulation study was carried out to compare the performances of the Bayes and maximum likelihood estimates at different sample sizes. Based on mean square error (MSE), the results reveal that the Bayes estimates performed better than maximum likelihood estimates. Based on posterior risk, Bayes estimates using Square Error Loss Function under the Extended Jeffery's Prior (SELFEXJ) performed best among the six non-informative priors considered under different sample sizes. Hence the Bayes estimate under the Extended Jeffrey's using the squared error loss function has the best estimator for estimating the shape parameter of the model.

Keywords: Bayesian Methods, Maximum Likelihood, Loss Functions, Posterior Risks, Mean Square Error.

INTRODUCTION

In the past, many generalized univariate continuous distributions have been proposed in literature. The generalization of these distributions is important in order to make their shape more flexible to capture the diversity present in the observed dataset. One of such generalizations is the Generalized Inverse exponential distribution (GIED) proposed by Abouammoh and Alshangiti (2009). In their work, they added shape parameter to make the distribution more flexible; and as a result, the parameter has to be estimated using the appropriate estimation technique. Maximum likelihood estimation and least squares estimation are used to estimate the parameters and reliability of the distribution Properties of the estimates are also studied. Krishna and Kumar (2013) studied Reliability estimation in generalized inverted exponential distribution with progressively type II censored sample, generalization of inverted exponential distribution was considered as a lifetime model.

Numerous researchers have estimated the parameters of different distributions using the Bayesian technique because of its advantage over other methods of estimation. Some of the researches include: the work of Farhad *et al.* (2013) studied the classical and Bayesian approach of estimating the scale parameter of Inverse Weibull distribution when the shape parameter was known under the assumption of quasi, gamma and uniform priors using square error loss function, entropy loss function and precautionary loss function. Dey (2015) studied Inverse Rayleigh Distribution

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using Bayesian estimation technique for the parameter estimates. Feroze (2012) discussed the Bayesian analysis of the scale parameter of inverse Gaussian distribution using different priors and loss functions. Yahgmaei *et al.,* (2013) proposed classical and Bayesian approaches for estimating the scale parameter in the Inverse Weibull Distribution when shape parameter is known.

Azam and Ahmed (2014) and Eraikhuemen, *et al.* (2020) respectively estimated the scale and shape parameter of Nakagami Distribution using Bayesian approach. The study revealed that the scale parameter was estimated under three prior distributions, namely; Uniform, Inverse Exponential and Levy priors and three loss functions namely; Squared Error Loss Function, Quadratic Loss Function and Precautionary Loss Function. Nasir *et al.,* (2015) studied Bayesian estimation of the scale parameter of log logistic distribution using square error loss function, precautionary loss function, simple precautionary loss function and weighted loss function with two non-informative priors (uniform and Jeffery priors). Kaisar *et al* (2016) studied the classical and Bayesian approach of scale parameter of Nakagami distribution under the assumption of Jeffrey, Extended Jeffrey and Quasi priors using quadratic, Al-Bayyati and entropy loss functions. The estimate of the scale parameter using simulated data set was obtained. Sanku Dey (2007) derived Bayes estimators for the parameters of inverted exponential distribution. These estimators are obtained on the basis of squared error and LINEX loss functions.

The focus of this paper is to apply the Bayesian method of estimating the shape parameter of generalized inverse exponential distribution under non-informative priors (Jeffery prior, uniform prior, extended Jeffery's prior) using two loss functions. Prior distributions play very crucial roles in Bayesian probability theory as it is attractive to have conditional distributions that have a closed form under sampling (Ogundeji *et al.,* 2018). An extensive Monte Carlo simulation was carried out to obtain and compare the performance of the different estimators for different sample sizes (n = 15, 35, 75 and 100) against different shape parameter *(β)* values of 0.5, 1.0, 1.5 and 2.0 with the assumption that the scale parameter is known. The Markov Chain Monte Carlo (MCMC) method is used to generate a process that moves through a large model space in order to adequately identify the high posterior probability models to average and generate parameter estimates (Ogundeji et al., 2022). The shape parameter of the Generalized Inverse Exponential Distribution (GIED) was also estimated using maximum likelihood and Bayesian estimation techniques. The Bayes estimates were obtained under the squared error function and precautionary loss function under the assumption of three non-informative priors.

The paper is organized as follows: section 2 defines the Generalized Inverse Exponential Distribution (GIED) model, the two loss functions applied, respective expressions for computations of maximum likelihood and Bayes estimates and their posterior risks. Section 3 contains Monte Carlo simulation analysis of the data, while section 4 consists of the discussion of the results and section 5 concludes the study.

MATERIALS AND METHODS

The Generalized Inverse Exponential Distribution, GIED (λ,) with cumulative density function (CDF) is expressed as follows (Sanjay *et al.,* 2013):

$$F(x) = 1 - \left(1 - e^{-\lambda / x}\right)^{\alpha}; \quad x \geq 0, \ \lambda > 0, \ \alpha > 0 \qquad (1)$$

and the probability density function (pdf) is given by:

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$$f(x)=\frac{\alpha\lambda}{x^2}e^{-\lambda/x}\left(1-e^{-\lambda/x}\right)^{\alpha-1};\quad x\geq 0,\ \lambda>0,\ \alpha>0 \quad (2)$$

The estimation of the shape parameter of generalized inverse exponential distribution using maximum likelihood and Bayesian estimation techniques is studied and implemented. Under the Bayesian approach, three non-informative priors (uniform prior, Jeffery's prior and extended Jeffery prior) were considered. Two loss functions (square error loss function and precautionary loss function) were considered in estimating the posterior distribution of the shape parameter (Sule and Adegoke, 2020):

The squared error loss function (SELF) is given as:

$$l_{sq}(\hat{\alpha}, \alpha) = c(\hat{\alpha}, \alpha)^2 \quad (3)$$

The risk function under square-error loss function is given as:

$$R_{RSQ}(\hat{\alpha}) = \int_0^\infty c(\hat{\alpha}-\alpha)^2 \pi_i(\alpha \mid x)\delta\alpha\ ;\ i = 1, 2, 3. \quad (4)$$

The Precautionary Loss Function (PLF) is given as:

$$l_{PLF}(\hat{\alpha}, \alpha) = \frac{c(\hat{\alpha}, \alpha)^2}{\hat{\alpha}} \quad (5)$$

The risk function under precautionary loss function is given as:

$$R_{RPLF}(\hat{\alpha}) = \int_0^\infty \frac{c(\hat{\alpha}-\alpha)^2}{\hat{\alpha}} \pi_i(\alpha \mid x)\delta\alpha\ ;\ i = 1, 2, 3. \quad (6)$$

After deriving the posterior distribution, the two loss functions, were employed to derive the estimators for the shape parameter. An efficient estimator is selected using the posterior risk and mean square error (MSE) criteria. Thus, the estimator with the minimum estimate is considered to be a better estimator of shape parameter of generalized inverse exponential distribution.

Estimation Methods of GIED

Maximum likelihood estimation (MLE)

Given the PDF of GIED in eqn (2) above, the maximum likelihood estimate is the value of the statistic which maximizes the likelihood function and it is obtained as (Abouammoh and Alshangiti, 2009):

$$\ln L(x;\alpha,\beta) = n\ln(\alpha) + n\ln(\beta) - \sum_{i=1}^{n}\ln\left(x_i^2\right) - \alpha\sum_{i=1}^{n}\frac{1}{x_i} + (\beta-1)\sum_{i=1}^{n}\ln\left(1-e^{\frac{\alpha}{x_i}}\right). \quad (7)$$

Differentiating eqn (7) with respect to β and setting it to zero:

$$\frac{d\ln L(x;\alpha,\beta)}{d\beta} = \frac{n}{\beta} + \sum_{i=1}^{n}\ln\left(1-e^{\frac{\alpha}{x_i}}\right) = 0. \quad (8)$$

Solving for β in eqn (7), gives:

$$\hat{\beta}_{MLE} = -\frac{n}{\sum_{i=1}^{n}\ln\left(1-e^{\frac{\alpha}{x_i}}\right)} \quad (9)$$

which can also be expressed as;

$$\hat{\beta}_{MLE} = \frac{n}{\sum_{i=1}^{n}\ln\left(1-e^{\frac{\alpha}{x_i}}\right)^{-1}} \quad (10)$$

Bayes Estimation (MCMC)

In Bayesian analysis, posterior distribution summarises what we know about uncertain quantities. It is a combination of the prior distribution and the likelihood function. Prior and posterior distributions play very crucial roles in Bayesian probability theory as it is attractive to have conditional distributions that have a closed form under sampling (Ogundeji and Adeleke, 2020; Aliyu and Abubakar, 2016). Bayesian estimation technique will be used to estimate the shape parameter of the GEID under three different priors and two loss functions.

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Square error loss function under the uniform prior (SELFU)

Bayes estimator of β relative to squared error loss function under uniform prior is obtained as:

$$\beta PR_{selfu} = \frac{(n+1)(n+2)}{N^2} - \left[\frac{(n+1)}{N}\right]^2 \tag{11}$$

The posterior risk is:

$$p(\beta/x) = \frac{\beta^n N^{n+1} e^{-N\beta}}{\Gamma(n+1)} \tag{12}$$

where $N = \left(\sum_{i}^{n} \ln\left[1 - e^{-\frac{\alpha}{x_i}}\right]^{-1}\right)$. (13)

Precautionary Loss Function under Uniform Prior (PLFU)

The Bayes estimator of β denoted by $\hat{\beta}_p$ relative to Precautionary loss function (PLF) is given as:

$$\hat{\beta}_p = \left[E(\beta^2/x)\right]^{\frac{1}{2}} \tag{14}$$

$$E(\beta^2/x) = \int_0^\infty \beta^2 p(\beta/x)d\beta \tag{15}$$

Expression (14) can be obtained by minimizing the expected loss $(E[L(\beta,\hat{\beta})])$ over β with respect to the posterior distribution $(p(\beta/x))$ i.e.

$$R(\beta,\hat{\beta}) = \int_0^\infty \frac{(\hat{\beta}-\beta)}{\hat{\beta}} P(\beta/x)d\beta$$

$$= \int_0^\infty (\hat{\beta}-\beta)^2 \hat{\beta}^{-1} P(\beta/x)d\beta \tag{16}$$

Integrating eqn (16), we obtained the posterior risk as:

$$BPR_{plfu} = \frac{2\left[\sqrt{(n+2)(n+1)} - (n+1)\right]}{N} \tag{17}$$

Square Error Loss Function under the Jeffrey's Prior (SELFJ)

Bayes estimator of β relative to squared error loss function under Jeffrey's prior is obtained as;

$$\hat{\beta}_{selfj} = \int_0^\infty \frac{\beta\beta^{n-1} N^n e^{-N\beta}}{\Gamma(n)} d\beta \tag{18}$$

Integrating eqn (18), w.r.t β we obtained the posterior risk as;

$$BPR_{selfj} = E(\beta^2/x) - [E(\beta/x)]^2 \tag{19}$$

Precautionary Loss Function under Jeffrey's Prior (PLFJ)

Bayes estimator of β relative to precautionary loss function under the Jeffrey's prior is obtained as;

$$\hat{\beta}_{plfj} = \left[\int_0^\infty \frac{\beta^2 \beta^{n-1} N^n e^{-N\beta}}{\Gamma(n)} d\beta\right]^{\frac{1}{2}} \tag{20}$$

The posterior risk is obtained as

$$BPR_{plfj} = \frac{2\left[\sqrt{n(n+1)} - (n)\right]}{N} \tag{21}$$

Square Error Loss Function under the Extended Prior (SELFEXJ)

Bayes estimator of β relative to squared error loss function under extended Jeffrey's prior is obtained as;

$$\hat{\beta}_{selfex} = \frac{n-2r+1}{N} \tag{22}$$

The posterior risk is obtained as:

$$\hat{\beta}_{selfex} = \frac{n-2r+1}{N^2} \tag{23}$$

Precautionary Loss Function under Extended Prior (PLFEXJ)

Bayes estimator of β relative to precautionary loss function under extended Jeffrey's prior is obtained as;

$$\hat{\beta}_{plfex} = \frac{\sqrt{(n-2r+1)(n-2r+2)}}{N} \tag{24}$$

The posterior risk is obtained as

$$\beta PR_{plfex} = 2\left[\frac{\sqrt{(n-2r+1)(n-2r+2)} - (n-2r+1)}{N}\right] \tag{25}$$

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RESULTS OF MARKOV CHAIN MONTE CARLO (MCMC) SIMULATION ANALYSIS

An extensive Monte Carlo simulation was carried out to obtain and compare the performance of the different estimators for different sample sizes (n = 15, 35, 75 and 100) against different initial values of the shape parameter *(β)* values of 0.5, 1.0, 1.5 and 2.0 with the assumption that the scale parameter is known. The Monte Carlo simulation were replicated 10,000 times and averaged over. To examine the performance of Bayes estimates for the shape parameter of GIED under the two loss functions, estimates are presented along with respective posterior risks and their MSC in Tables below. The shape parameter of the Generalized Inverse Exponential Distribution (GIED) was also estimated using maximum likelihood (ML). Under the Bayes estimates were six estimators using the squared error function and precautionary loss function under the assumption of three non-informative priors. The six different Bayes estimators that were considered are: Square Error Loss Function under the Uniform Prior (SELFU), Square Error Loss Function under the Jeffrey's Prior (SELFJ), Square Error Loss Function under the Extended Jeffrey's Prior (SELFEXJ), Precautionary Loss Function under Uniform Prior (PLFU), Precautionary Loss Function under Jeffrey's Prior (PLFJ), and Precautionary Loss Function under Extended Jeffrey's Prior (PLFEXJ). Tables 1, 2, 3 and 4 show the Monte Carlo simulation results based on these estimators by way of simulation.

Table 1: Maximum Likelihood and Bayes Average Estimates of Shape Parameter (asterisk), Posterior Risk (Parenthesis) and MSE (Bolded) for n = 15

n	Method	β = 0.5	β = 1.0	β = 1.5	β = 2.0
	ML	0.5349772*	1.069954*	1.604932*	2.139909*
		0.02673302	**0.10693201**	**0.24059733**	**0.42772846**
15	**Bayes:**				
	(i) SELFU	0.5706424*	1.141285*	1.711927*	2.282569*
		(0.02191433)	(0.08765733)	(0.197229)	(0.3506293)
		0.0393622	**0.1574489**	**0.3542597**	**0.6297949**
	(ii) SELFJ	0.5349772*	1.069954*	1.604932*	2.139909*
		(0.02054469)	(0.08217874)	(0.1849022)	(0.328715)
		0.02673302	**0.10693201**	**0.24059733**	**0.42772846**
	(iii) SELFEXJ	0.4279818*	0.8559636*	1.283945*	1.711927*
		(0.01643575)	(0.06574299)	(0.1479217)	(0.262972)
		0.01408988	**0.05635951**	**0.12680883**	**0.22543800**
	(iv) PLFU	0.5882047*	1.1764090*	1.7646140*	2.352819*
		(0.03512464)	(0.07024929)	(0.1053739)	(0.1404986)
		0.42772846	**0.19051321**	**0.42865497**	**0.76205349**
	(v) PLFJ	0.5525221*	1.1050440*	1.6575660*	2.210088*
		(0.03508975)	(0.07017951)	(0.1052693)	(0.140359)
		0.03222176	**0.12888698**	**0.28999571**	**0.51554793**
	(vi) PLFEXJ	0.4454576*	0.8909151*	1.336373*	1.78183*
		(0.03495156)	(0.06990312)	(0.1048547)	(0.1398062)
		0.01445734	**0.05782933**	**0.13011607**	**0.23131728**

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Table 2: Maximum Likelihood and Bayes Average Estimates of Shape Parameter (asterisk), Posterior Risk (Parenthesis) and MSE (Bolded) for n = 35.

n	Method	β = 0.5	β = 1.0	β = 1.5	β = 2.0
	ML	0.5143571	1.028714	1.543071	2.057429
		0.008724456	**0.034897810**	**0.078520071**	**0.139591374**
35	**Bayes:**				
	(i) SELFU	0.5290531*	1.058106*	1.587159*	2.116212*
		(0.00800992)	(0.03203968)	(0.07208928)	(0.1281587)
		0.01047743	**0.04190972**	**0.09429688**	**0.16763889**
	(ii) SELFJ	0.5143571*	1.028714*	1.543071*	2.057429*
		(0.00778742)	(0.03114969)	(0.0700868)	(0.1245987)
		0.008724456	**0.034897810**	**0.078520071**	**0.139591371**
	(iii) SELFEXJ	0.4702694*	0.9405388*	1.410808*	1.881078*
		(0.007119929)	(0.02847971)	(0.06407936)	(0.1139189)
		0.006701615	**0.026806461**	**0.060314521**	**0.107225891**
	(iv) PLFU	0.5363507*	1.0727010*	1.6090520*	2.1454030*
		(0.01459526)	(0.02919051)	(0.04378577)	(0.05838103)
		0.0115956	**0.04638238**	**0.10436042**	**0.18552970**
	(v) PLFJ	0.5216534*	1.0433070*	1.564960*	2.0866130*
		(0.01459242)	(0.02918484)	(0.04377726)	(0.05836968)
		0.009515161	**0.038060660**	**0.085636431**	**0.152242495**
	(vi) PLFEXJ	0.4775608*	0.9551217*	1.4326820*	1.9102430*
		(0.01458287)	(0.02916573)	(0.0437486)	(0.05833147)
		0.006759375	**0.027037507**	**0.060834343**	**0.108149982**

Table 3: Maximum Likelihood and Bayes Average Estimates of Shape Parameter (asterisk), Posterior Risk (Parenthesis) and MSE (Bolded) for n = 75

n	Method	β = 0.5	β = 1.0	β = 1.5	β = 2.0
	ML	0.5064477	1.012895	1.519343	2.025971
		0.003655993	**0.014623960**	**0.032903932**	**0.058495899**
75	**Bayes:**				
	(i) SELFU	0.5132003*	1.0264010*	1.5396010*	2.0528010*
		(0.00351191)	(0.01404767)	(0.03160727)	(0.0561907)
		0.003997911	**0.015991655**	**0.035981200**	**0.063966556**
	(ii) SELFJ	0.5064477*	1.0128950*	1.519343*	2.0257910*
		(0.00346570)	(0.01386284)	(0.03119138)	(0.05545134)
		0.003655993	**0.014623960**	**0.032903932**	**0.058495899**
	(iii) SELFEXJ	0.4861898*	0.9723796*	1.458569*	1.944759*
		(0.003327081)	(0.01330832)	(0.02994373)	(0.05323329)
		0.003238089	**0.012952357**	**0.029142788**	**0.051809418**
	(iv) PLFU	0.5165656*	1.033131*	1.549697*	2.066262*
		(0.00673056)	(0.01346114)	(0.0201917)	(0.02692227)
		0.004210489	**0.016841948**	**0.037894408**	**0.067367793**
	(v) PLFJ	0.5098128*	1.019626*	1.529438*	2.039251*
		(0.00673027)	(0.01346055)	(0.02019083)	(0.0269211)
		0.003812596	**0.015250396**	**0.034313347**	**0.061001525**
	(vi) PLFEXJ	0.4895545*	0.9791089*	1.468663*	1.958218*
		(0.006729351)	(0.0134587)	(0.02018805)	(0.0269174)
		0.003249865	**0.012999459**	**0.029248768**	**0.051997846**

Table 4: Maximum Likelihood and Bayes Average Estimates of Shape Parameter (asterisk), Posterior Risk (Parenthesis) and MSE (Bolded) for n = 100.

n	Method	$\beta = 0.5$	$\beta = 1.0$	$\beta = 1.5$	$\beta = 2.0$
	ML	0.5049245	1.009849	1.514774	2.019698
		0.002680352	**0.010721407**	**0.024123181**	**0.042885627**
100	**Bayes;**				
	(i) SELFU	0.5099738*	1.019948*	1.529921*	2.039895(
		(0.00260078)	(0.01040314)	(0.02340706)	(0.04161255)
		0.002868258	**0.011473041**	**0.025814308**	**0.045892118**
	(ii) SELFJ	0.5049245*	1.009849*	1.514774*	2.019698*
		(0.00257503)	(0.01030014)	(0.02317531)	(0.04120054)
		0.002680352	**0.010721407**	**0.024123181**	**0.042885627**
	(iii) SELFEXJ	0.4897768*	0.9795536(	1.469330*	1.959107*
		(0.002497783)	(0.009991132)	(0.02248005)	(0.03996453)
		0.002447768	**0.009791073**	**0.022029903**	**0.039164286**
	(iv) PLFU	0.5124922*	1.024984*	1.537477*	2.049969*
		(0.00503680)	(0.01007362)	(0.01511043)	(0.02014724)
		0.002984406	**0.011937613**	**0.026859665**	**0.047750500**
	(v) PLFJ	0.5074429*	1.014886*	1.522329*	2.029772*
		(0.00503668)	(0.01007337)	(0.01511006)	(0.02014674)
		0.002766707	**0.11066834**	**0.024900377**	**0.044267337**
	(vi) PLFEXJ	0.492295*	0.9845899*	1.4768850*	1.969180*
		(0.005036299)	(0.0100726)	(0.0151089)	(0.02014519)
		0.022088708	**0.009817201**	**0.022088708**	**0.039268814**

DISCUSSION OF RESULTS

As expected, it was observed that the performance of both the maximum likelihood estimates (MLE) and the Bayes estimates improve as the sample sizes increases. Also, the maximum likelihood estimates (MLE) and Bayes estimates becomes closer as the sample size increases. It was observed that based on mean square errors (MSE) only, the SELFEXJ and PLFEXJ Bayes estimates performed better than maximum likelihood estimates. However, the SELFJ Bayes estimates are equally efficient as the Maximum Likelihood (ML) estimates having same MSE.

When compared in terms of mean square errors (MSE) and posterior risk, the estimates were better at smaller values of β. Hence the estimate is better at small value of $\beta = 0.5$. The Extended Jeffrey's prior tends to perform better than the uniform and Jeffrey's priors when compared in terms of their mean square errors (MSE) under both loss functions used.

The uniform prior under the Square Error Loss Function (SELF) was observed to have better estimate than the uniform prior under the PLF at all sample sizes. The Extended Jeffrey's prior under the Square Error Loss Function (SELF) was observed to have performed better than the estimate of Extended Jeffrey's prior under the Precautionary Loss Function (PLF). Furthermore, the estimate of Jeffrey's prior under the Square Error Loss Function (SELF) performed better than the estimate of Jeffrey's prior under the Precautionary Loss Function (PLF).

Based on mean square error (MSE), the results reveal that the Bayes estimates performed better than maximum likelihood estimates. It can also be observed that among all the Bayes estimates, the Square Error Loss Function (SELF) under the Extended Jeffrey's

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prior performed best compared with the other estimates, since Square Error Loss Function (SELF) under the Extended Jeffrey's prior have the minimum posterior risk and mean square error.

As shown in graphs of mean square error against sample size at different values of the shape parameter (Figures 1, 2, 3 and 4), the square error loss function under the extended Jeffrey's prior which is the black line have the best estimates for all the values of β used.

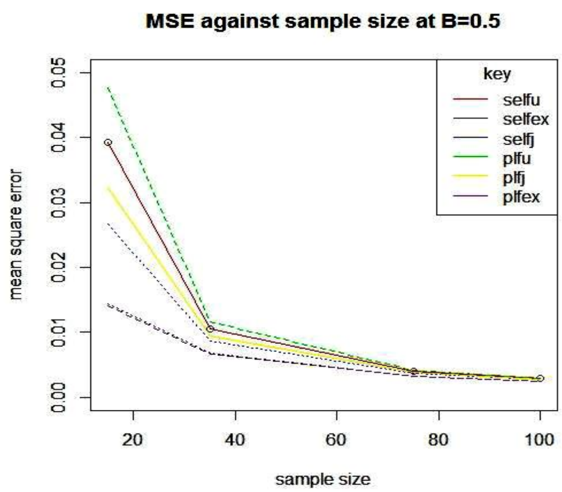

Figure 1: Graphs of MSE against different sample sizes of the Bayes estimates at $\beta = 0.5$

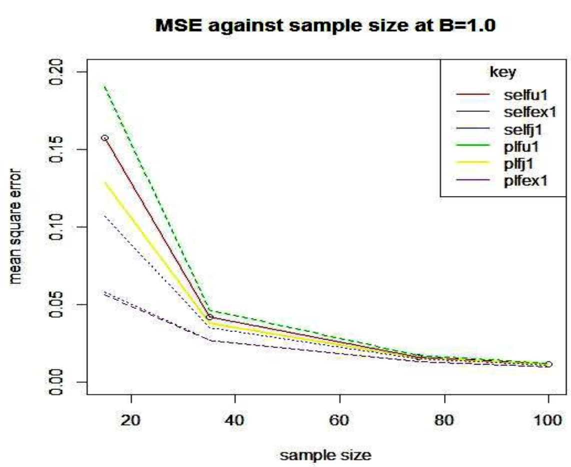

Figure 2: Graphs of MSE against different sample sizes of the Bayes estimates at $\beta = 1.0$

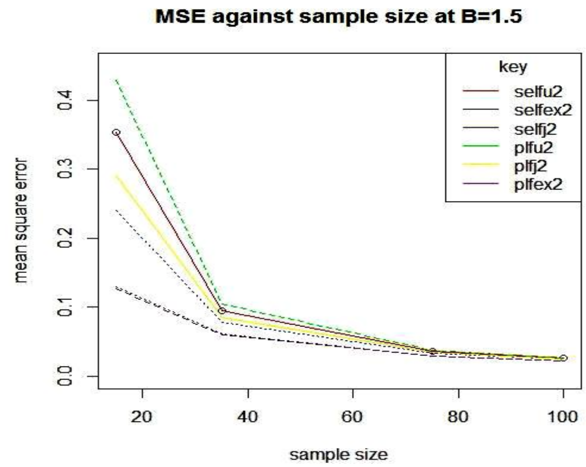

Figure 3: Graphs of MSE against different sample sizes of the Bayes estimates at $\beta = 1.5$

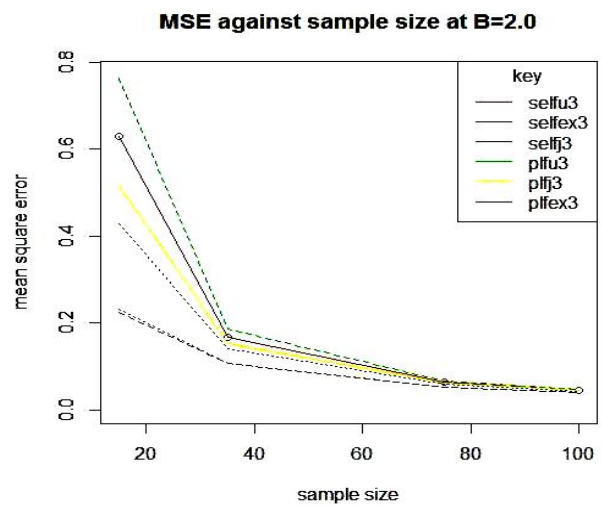

Figure 4: Graphs of MSE against different sample sizes of the Bayes estimates at $\beta = 2.0$

Also, shown graphically below (Figures 5, 6, 7 and 8) are the 99% confidence bound of the estimates obtained are from the Generalized Inverse Exponential Distribution (GIED). It is observed the actual shape parameter is within the estimated confidence bounds.

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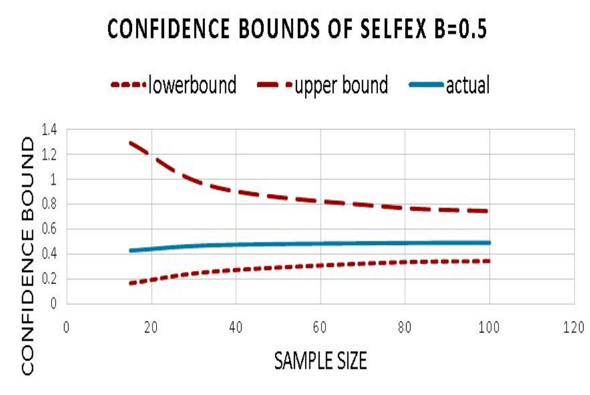

Figure 5: Graphs of the 99% confidence bound against different sample sizes of the Bayes estimates at β = 0.5

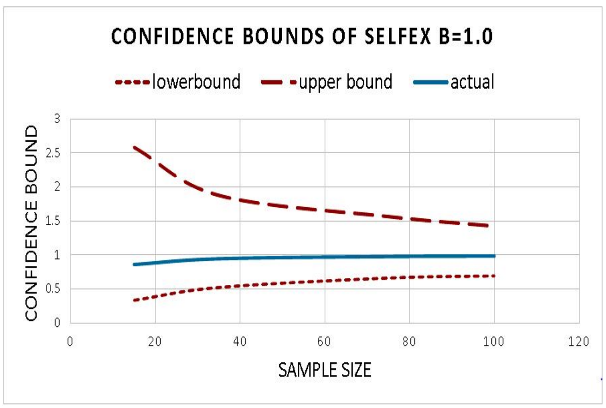

Figure 6: Graphs of the 99% confidence bound against different sample sizes of the Bayes estimates at β = 1.0

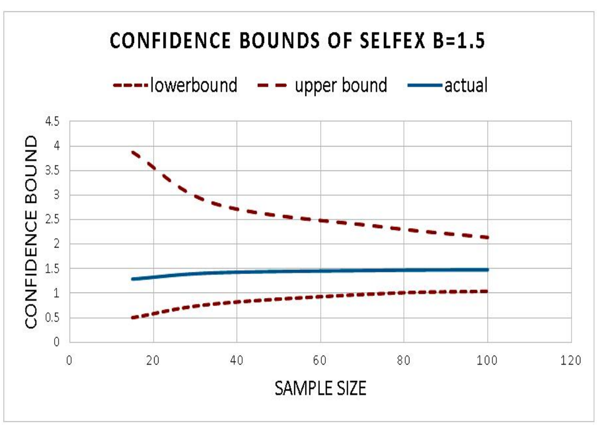

Figure 7: Graphs of the 99% confidence bound against different sample sizes of the Bayes estimates at β = 1.5

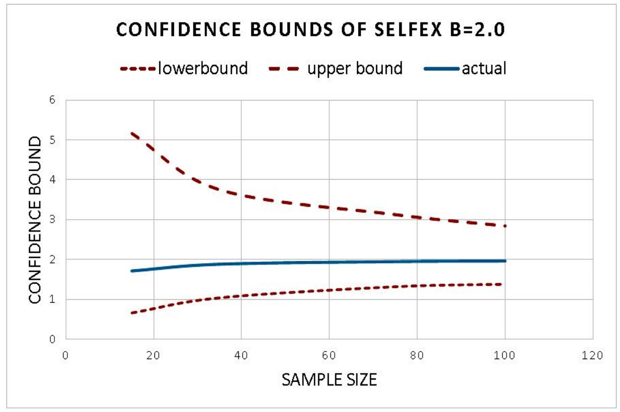

Figure 8: Graphs of the 99% confidence bound against different sample sizes of the Bayes estimates at β = 2.0

CONCLUSION

From the result of the analysis, it was concluded;

i. The estimates become better as the sample size increases and are better at smaller value of the shape parameter *(β)*.

ii. Based on mean square errors (MSE) only, the SELFEXJ and PLFEXJ Bayes estimates performed better than maximum likelihood estimates. However, the SELFJ Bayes estimates are equally efficient as the Maximum Likelihood (ML) estimates having same MSE.

iii. Among the non-informative priors considered, the extended Jeffrey's prior gives the best estimates compared with the uniform and Jeffrey's priors based on the posterior risks.

iv. Among all the Bayes estimates, the Square Error Loss Function (SELF) under the Extended Jeffrey's prior performed best compared with the other estimates, since Square Error Loss Function (SELF) under the Extended Jeffrey's prior have the minimum posterior risk and mean square error.

Therefore, the Square Error Loss Function (SELF) under the extended Jeffrey's prior has an efficient estimator for estimating the shape parameter of the Generalized Inverse

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Exponential Distribution. The outcomes of this study is an improvement on the work of Sule. and Adegoke (2020) and consistent with the work of other authors in the same areas, (Sanjay *et al.,* 2013; Krishna and Kumar, 2013).

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