PARADIGM SHIFT IN MAIN MACROECONOMIC VARIABLES' ANALYSIS

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ABSTRACT

Bayesian Vector Autoregressive (BVAR) models are mostly used in computational analysis of macroeconomic variables of a nation. Certain issues had led to imprecise estimation by BVAR Model. Recently, it has been discovered that BVAR exhibit some statistical imprecision and inaccuracy in evaluating and forecasting macroeconomic variables. Because of this inconsistency with VAR models, the focus has gradually shifted to models that can account for the co-movement among macroeconomic variables. This shift is occasioned by the limitations associated with BVAR and its inability to elucidate information relating to the integration and cointegration among the macroeconomic variables. Also, the predictive accuracy of BVAR model has some blemish owing to the fact that it lacks the estimation capacity to elucidate vital information about the co-movement among macroeconomic variables of a developing economy like Nigeria. The research utilizes secondary data of the selected macroeconomic variables which was obtained from the statistical bulletin of the Central Bank of Nigeria (CBN) ranging from 1986 to 2019. Bayesian Vector Error Correction Model (BVECM) was used in the analysis of the macroeconomic variables. It was discovered among others that BVECM has more analytical features than BVAR. Therefore, in forecasting and evaluating main macroeconomic variables of a nation, BVECM should be considered because it possesses a better predictive ability than the BVAR model since it predicts the direction of change in the chosen macroeconomic variables.

Keywords: Paradigm Shift, BVAR, BVECM, Overparameterization, hyperparameter

INTRODUCTION

Paradigm shift is a major change in practice or concept in accomplishing a task. Conventionally, macroeconomic variables are evaluated, estimated and forecasted using the Vector Autoregressive model (VAR) and its extension in the Bayesian realm: Bayesian Vector Autoregressive Model (BVAR) (Hapsari et al, 2020). In most cases, the application of VAR models relies on the linearity of the variables and their lags. This linearity assumption over times do lead to inconsistent, imprecise and spurious results and have a negative impact on the forecasting accuracy of the models (Huber and Rossini, 2020). Also, BVAR has a problem of overparameterization, a situation in which the number of estimated parameters is higher than the amount of data used in a study (Hapsari, et al, 2021). The constant changing of error covariance matrix of BVAR model also contribute to over parameterization. Misspecification associated with BVAR also make its forecasting imprecise and sometime spurious (Koop and Korobilis, 2010).

Despite the theoretical attractiveness of BVAR, its results in most studies do not agree about the nature of the hypothetical gains from incorporating cointegrating relationships. LeSage as quoted by Hapsari et al (2020) looked at the forecasting performance of BVECM and BVAR model and affirmed that the superiority of BVECM over BVAR at increasing forecasting horizons. It also has the problem of overfitting and parameter instability which result in poor out-of-sample forecasting performance. These weaken the forecasting ability of the model and the model unsuitable for the estimation. Also, when the lag is large, BVAR leads to over fitting of the macroeconomic data resulting to imprecise model forecast (Koop and Korobilis, 2013). Because of this inconsistency with VAR models, the focus has gradually shifted to models that accommodate and account for co-movement and time variation in the parameters (Koop and Korobilis, 2010).

Bayesian models are data analytical process derived from the principles and pattern of Bayesian inference. It entails statistical estimations having good statistical properties which is anchored on computational frameworks use in model estimation, selection and statistical validation. Bayesian estimation and analytical methods are anchored on the posterior distribution that evolved from the statistical combination of prior distribution and likelihood function. For better quality and advancement of nation through the macroeconomic variables, forecasting and modelling is the central theme of econometrics and by extension Bayesian econometrics (Carriero, et al, 2019). In statistics, forecasting helps to unveiled what the future holds. This has motivated researchers to developed statistical models for efficient and effective forecasting of macroeconomic variables. Forecasting of macroeconomic variables of a nation is central to the policy, decision and investment process and as well as all the financial planning of the nation.

In statistical analysis, the long term comovement or cointegration and relationship among macroeconomic variables may be desirable. These cannot be obtained from the VAR and BVAR models. These and more have necessitated the Paradigm Shift from BVAR to and BVECM. BVECM is an error correction extension of the BVAR for forecasting and predicting macroeconomic variables. It is the application or addition of Bayesian components to the Vector Error Correction Models (VECMs) (Adenomon, 2017). In the evaluation of macroeconomic variables, equilibrium relationships are desired among many macroeconomic variables. This is achieved through the imposition of restrictions on the BVECMs' parameters that increases the estimation precision of the parameters. A BVAR model with Error Correction Mechanism (also known as BVECM) can be used to combine BVAR model's advantages with the benefits of taking into cognizance the explicitly long run relationship in forecasting macroeconomic variables of a nation. BVECM possess more variable explanatory ability than the BVAR models. Besides, BVECM out of sample forecast are systematically unbiased and produces more efficiency during statistical analysis. Furthermore, BVECM attracts a long run additional information when predicting macroeconomic variables (Chen and Lueng, 2003). So, BVECM is economically significant when predicting the directional changes associated with macroeconomic variables. BVECM are used in an economy where the financial predictability is unreliable and inconsistent.

The application of Bayesian methods to the analysis and evaluation of vector error correction model results in a better and improved model termed the Bayesian Vector Error Correction Model (BVECM). Cross and Poon (2016) rent their support to the superiority of Bayesian Vector Error Correction Model (BVECM) over Bayesian Vector Autoregressive (BVAR) model when used with the Minnesota or Litterman prior and most especially when the macroeconomic variables are integrated and co-integrated. The BVECM has the ability to adjust back to equilibrium state when shock is experience. Chen and Leung (2003) explored the BVECM for forecasting the exchange rates of three major Asia countries (Korea, Japan and Australia). These country's economy were chosen because of the reliance of other economy on their performance. Therefore, financial instability of any of these economy has direct bearing on other countries of the region. The exchange rates used are Korea Won/US dollar, Japan Yen/US dollar and Australia dollar/US dollar exchange rates. It was discovered from the study that BVECM outperformed BVAR in forecasting of the exchange rates of these countries. Besides, BVECM produces a better out of sample forecast than BVAR.

One of the core duty of the government of a nation is to formulate macroeconomic policies which helps in sustaining viable economic growth having a good economic variable. In all firmness, a nation's macroeconomic stability and sustainability is anchored on viable Gross Domestic Product (GDP) and this has great influence on other macroeconomic variables. GDP serves as the 'main stream' which other tributaries flows into as far as the economy of a nation is concern. It is also characterizing as the backbone of the economy. Onwukwe and Nwafor (2014) lend their voices among many other scholars in supporting the fact that the key macroeconomic variables in Nigeria are GDP, Inflation rate and Unemployment rate. Although, exchange rate and interest rate are at the corridor of main macroeconomic variables. Therefore, this study intends to investigate the superiority of BVECM and BVAR in macroeconomic variables' analysis.

MATERIALS AND METHODS

The research utilizes secondary data of the selected macroeconomic variables (Real Gross Domestic Product (GDP), Inflation Rate and Unemployment Rate) which was obtained from the statistical bulletin of the Central Bank of Nigeria (CBN) ranging from 1986 to 2019. R-statistical software (version R-3.3.3) was used in the analysis.

Prior

The prior considered in this research natural conjugate prior which takes the form of Normal Inverse Gamma (NIG) Distribution. The Natural Conjugate prior also known as the convenience prior is a type of prior when combined with the likelihood function gives a posterior distribution that belong to the same family (Koop, 2003). The natural conjugate prior and the posterior distribution possess similar functional form (Koop, 2003). The natural conjugate prior gives useful and reliable macroeconomic analytical results. Besides, with the presence of posterior covariance matrix of BVAR coefficient, it speeds up computational analysis and facilitate simulation when used with natural conjugate prior (Chan, 2019).

Normal Inverse Gamma Prior Distribution

The Normal-Inverse Gamma prior distribution is a type of natural conjugate prior (asymmetric conjugate prior) that permits the differential treatment of prior variances of own lags against the lags of others.

Given a pair ie (θ_i, σ^2) such that $i = 1,2,..n$; then the NIG prior density function is as follows;

$$pr(\theta, \sigma^2) = \prod_{i=1}^{n} k\left(\sigma_i^2\right)^{-(c_i+1+\frac{v_i}{2})} exp^{-\frac{1}{\sigma_i^2}\left[S_i+\frac{1}{2}(\theta_i-d_i)'\beta_i^{-1}(\theta_i-d_i)\right]} \quad (1)$$

Where $k = (2\pi)^{-\frac{v_1}{2}}|\beta|^{-\frac{1}{2}}S_i^{c_i}/\Gamma(c_i)$

θ_i is the prior variance, (d_i, β_i, c_i, S_i) such that $i = 1,2,..n$; are the hyperparameter of the asymmetric conjugate prior. Also, the prior covariance matrix β_i induces shrinkage in the prior distribution.

Furthermore,
$\left(\theta_i \setminus \sigma_i^2\right) \sim N\left(d_i, \sigma_i^2, \beta_i\right)$, $\sigma_i^2 \sim IG(c_i, S_i)$, this implies that;
$\left(\theta_i \setminus \sigma_i^2\right) \sim NIG(d_i, \beta_i, c_i, S_i)$ (Chan, 2019). (2)

Posterior Distribution for Normal Inverse Gamma Prior Distribution

The Normal Inverse Gamma (NIG) which is also known as the Gaussian Inverse Gamma distribution belong to the family of multivariate continuous probability distribution having four parameters and NIG is the conjugate prior for multivariate normal distribution having unknown mean and variance.

In a concise form, the NIG prior is given as;

$$p(\mu, \sigma^2) = NIG(m_0, V_0, \theta_0, \beta_0)$$
$$= \mathcal{N}(\mu|m_0, \sigma^2 V_0) IG(\sigma^2|\theta_0, \beta_0)$$

Given the likelihood function of VECM as (3) and the asymmetric natural conjugate prior distribution of equation (2), the resultant posterior distribution is the combination of these equations in (4) which is as follows;
The likelihood function of VECM as:

$$L(D|\Pi, \Gamma, \textstyle\sum) =$$

The posterior distribution of $\left(\theta_i, \sigma_i^2\right)$ for $i = 1,2, .. n$; is;

$$pr(\theta_i, \sigma^2/y) \propto pr(\theta, \sigma^2) pr(y/\theta, \sigma^2)$$

$$pr(\theta_i, \sigma^2/y) = \prod_{i=1}^{n} pr\left(\theta_i, \sigma_i^2\right) pr\,(y/\theta, \sigma^2)$$

$$pr(\theta_i, \sigma^2/y) = \prod_{i=1}^{n} k_i (2\pi)^{-\frac{n}{2}} \left(\sigma_i^2\right)^{-(c_i + \frac{n+k_i}{2} + 1)} exp^{-\frac{1}{\sigma_i^2}\left[\bar{S} + \frac{1}{2}(\theta_i - \bar{\theta}_i)' K_{\theta_i} (\theta_i - \bar{\theta}_i)\right]} \quad (3)$$

Where $K_{\theta_i} = \beta_I^{-1} + X_I' X_i$, $\overline{\theta_i} = K_{\theta_i}^{-1}(\beta_I^{-1} d_i + X_I^{-1} y_i)$ and $\bar{S} = S_i + (y_i' y_i + d_i' \beta_i^{-1} d_i - \overline{\theta_i'} K_{\theta_i} \theta_i) / 2$

In a compact form, the posterior distribution is expressed as;

$$\left(\theta_i, \sigma_i^2 \setminus y\right) \sim NIG(\bar{\theta}_i, K_{\theta_i}^{-1}, c_i + \frac{n}{2}, \bar{S}_i)$$

Such that $i = 1,2, .. n$.

This shows that the posterior distribution is also a Normal Inverse Gamma (a product of n-NIG distribution). Also, from the properties of NIG distribution, the posterior mean and variance can be obtained.

Posterior Predictive for Normal Inverse Gamma Prior Distribution

The posterior predictive for the Normal Inverse Gamma Prior Distribution is given as;

$$p(x|y) = \iint p(x|\mu, \sigma^2) p(\mu, \sigma^2|y) d\mu d\sigma^2$$
$$= \frac{p(x|y)}{p(y)}$$
$$= \frac{\Gamma[(v_n + 1)/2]}{\Gamma(v_n/2)} \sqrt{\frac{k_n}{k_n + 1}} \frac{(v_n \sigma_n^2)^{v_n/2}}{\left[v_n \sigma_n^2 + \frac{k_n}{k_n + 1}(x - \mu_n)^2 (v_n + 1)/2\right]} \frac{1}{\pi^{1/2}}$$

$$= \frac{\Gamma[(v_n+1)/2]}{\Gamma(v_n/2)}\left[\frac{k_n}{(k_n+1)\pi v_n\sigma_n^2}\right]^{\frac{1}{2}}\left[1+\frac{k_n}{(k_n+1)v_n\sigma_n^2}\right]^{-(v_n+1)/2}$$

In a simplified form, the above equation becomes;

$$t_{v_n}(\mu_n, \frac{(1+k_n)\sigma_n^2}{k_n})$$

It is sufficing to say that at this point, the posterior predictive are;

$$\frac{k_n}{(1+k_n)\sigma_n^2} = \frac{1}{(\frac{1}{k_n}+1_n)\sigma_n^2}$$

$$= \frac{2\alpha_n}{2\beta_n(1+V_n)}$$

$$= \frac{\alpha_n}{\beta_n(1+V_n)}$$

Therefore, the resultant posterior predictive is;

$p(y|x) = t_{2\alpha_n}(m_n, \frac{\beta_n(1+V_n)}{\alpha_n})$ (Murphy, 2007, Chan, 2019). (4)

Marginal Likelihood

The analytical expression of marginal likelihood helps to obtain the optimal hyperparameters of the prior distribution. For the avoidance of mathematical ambiguity, it is expedient to compute the marginal likelihood in logarithms scale. The analytical expression of marginal likelihood of VAR(p) is given as follows; combining the likelihood function of equation (3) and NIG prior of equation (1), the resultant marginal likelihood is given as;

$$pr(y) = (2\pi)^{-\frac{nt}{2}}\left|\Omega K_{\theta_i}\right|^{-\frac{n}{2}}\frac{\Gamma_n(\frac{v_0+t}{2})|\underline{S}|^{\frac{v_0}{2}}}{\Gamma_n(\frac{v_0}{2})|\bar{S}|^{\frac{v_0+t}{2}}} \quad (5)$$

Where K_{θ_i} is the precision matrix (Chan et al, 2019).

The log marginal likelihood is;

$logpr(y) =-\frac{nt}{2}\log(2\pi) + \sum_{i=1}^{n}\left[-\frac{1}{2}(log\Omega + \log K_{\theta_i}) + \log\Gamma_n(\frac{v_0+t}{2}) + \frac{v_0}{2}log\underline{S} - log\Gamma_n(\frac{v_0}{2}) - \frac{v_0+t}{2}log|\bar{S}|\right]$ (Chan, 2019). (6)

The conditional likelihood of the VAR(p) in equation (10) can be written as:

$(\theta_i|y, \sigma_i^2) \sim N(\hat{\theta}_i, \sigma_i^2\boldsymbol{A}_{\theta_i}^{-1})$ (7)

where $\sigma_i^2\boldsymbol{A}_{\theta_i}^{-1}$ is the covariance matrix (Carriero, et al, 2019, Chan, 2019).

Marginal Likelihood of Normal Inverse Gamma Prior distribution

Given a VAR (p) under the asymmetric natural conjugate prior (NIG), the Marginal Likelihood is derived as follows:

$$p(y) = \prod_{i=1}^{n}\int p(\theta_i,\sigma_i^2)\, p\left(y_i\middle|(\theta_i,\sigma_i^2)\right) d(\theta_i,\sigma_i^2)$$

$$= \prod_{i=1}^{n} c_i(2\pi)^{-\frac{T}{2}}\int (\sigma_i^2)^{-(v_i+\frac{T+k_i}{2}+1)} e^{-\frac{1}{\sigma_i^2}\left[\left[\hat{S}_i+\frac{1}{2}(\theta_i-\hat{\theta}_i)K_{\theta_i}(\theta_i-\hat{\theta}_i)\right]\right.} d(\theta_i,\sigma_i^2)$$

$$= \prod_{i=1}^{n} c_i (2\pi)^{-\frac{T}{2}} (2\pi)^{\frac{k_i}{2}} |\boldsymbol{K}_{\theta_i}^{-1}|^{\frac{1}{2}} \frac{\Gamma(\nu_i + {}^{T}/_{2})}{S_i^{\nu_i + \frac{T}{2}}}$$

$$p(y) = \prod_{i=1}^{n} (2\pi)^{-\frac{T}{2}} |\boldsymbol{V}_i|^{\frac{-1}{2}} |\boldsymbol{K}_{\theta_i}^{-1}|^{\frac{-1}{2}} \frac{\Gamma(\nu_i + {}^{T}/_{2}) S_i^{\nu_i}}{\Gamma(\nu_i) S_i^{\nu_i + \frac{T}{2}}}$$

Bayesian Vector Error Correction Model (BVECM)

Because of the presence of stochastic trend, most economic variables are often not stationary at first level, but become stationary at first difference. In such situation, we employ model that have the error correction mechanism. Vector Error Correction Models (VECMs) belong to a category of multiple time series models most commonly used for parameter whose underlying variables have a long-run stochastic trend. This long-run stochastic trend is also called co-integration. In order to apply the BVECM, the variables must be integrated and there must be an existence of cointegration among the macroeconomic variables The Bayesian Vector Error Correction Model (BVECM) has the tendency to produce a better forecast than BVAR by taking information from the long run.

VECM having lag of endogenous variables p with cointegration rank is given as;

$$\Delta Y_t = \Pi Y_{t-1} + \Gamma_1 \Delta Y_{t-1} + \Gamma_2 \Delta Y_{t-2} + \ldots + \Gamma_p \Delta Y_{t-p} + \Gamma_p \Delta Y_{t-p+1} + \varepsilon_t \quad (8)$$

Given the prior distribution in equation (4) and the likelihood function:

$$L(x|\Pi, \Gamma, \Sigma) = f(Y_1) f(Y_2) f(Y_3) \ldots f(Y_N)$$

$$L(\boldsymbol{x}|\boldsymbol{\Pi}, \boldsymbol{\Gamma}, \boldsymbol{\Sigma}) = \prod_{n=1}^{N} \frac{1}{2\pi^{k/2} \Sigma^{1/2}} \exp -\frac{1}{2} \left[(\Delta Y - \Pi' Y_{-1} - \Gamma \Delta X) \Sigma_u^{-1} (\Delta Y - \Pi' Y_{-1} - \Gamma \Delta X)' \right]$$

$$L(\boldsymbol{x}|\boldsymbol{\Pi}, \boldsymbol{\Gamma}, \boldsymbol{\Sigma}) = \prod_{n=1}^{N} \frac{1}{2\pi^{kN/2} \Sigma^{N/2}} \exp -\frac{1}{2} \left[(\Delta Y - \Pi' Y_{-1} - \Gamma \Delta X) \Sigma_u^{-1} (\Delta Y - \Pi' Y_{-1} - \Gamma \Delta X)' \right]$$

$$L(\boldsymbol{x}|\boldsymbol{\Pi}, \boldsymbol{\Gamma}, \boldsymbol{\Sigma}) = \prod_{n=1}^{N} 2\pi^{-kN/2} \Sigma^{-N/2} \exp -\frac{1}{2} tr \left[\Sigma_u^{-1} \left(\Delta Y - \Pi' Y_{-1} - \Gamma \Delta X \right) \left(\Delta Y - \Pi' Y_{-1} - \Gamma \Delta X \right)' \right]$$

In a compact form and omitting the normalizing constant, as shown by Hapsari et al (2021),

$$\boldsymbol{L}(\boldsymbol{x}|\boldsymbol{\Pi}, \boldsymbol{\Gamma}, \boldsymbol{\Sigma}) = |\boldsymbol{\Sigma}|^{-N/2} exp \left[-\frac{1}{2} tr(\boldsymbol{\Sigma}^{-1} \mathbf{A}' \mathbf{A}) \right] \quad (9)$$

The log likelihood function of BVECM is;

$$Inl = -\frac{kN}{2} In 2\pi - \frac{N}{2} In|\Sigma_u| - tr\left[\Sigma_u^{-1} (\Delta Y - \Pi' Y_{-1} - \Gamma \Delta X)(\Delta Y - \Pi' Y_{-1} - \Gamma \Delta X)' \right] \quad (10)$$

Therefore, the joint posterior distribution for BVECM parameters is shown below:

$$f\big((\boldsymbol{\Pi}, \boldsymbol{\Gamma}, \boldsymbol{\Sigma}|\boldsymbol{x})\big) \propto \boldsymbol{L}(\boldsymbol{x}|\boldsymbol{\Pi}, \boldsymbol{\Gamma}, \boldsymbol{\Sigma}) \times f(\boldsymbol{\Pi}, \boldsymbol{\Gamma}, \boldsymbol{\Sigma}) \text{ (Hapsari et al, 2021)} \quad (11)$$

The general Bayesian Vector Error Correction Model (BVECM) variable y_t is as follows:

$$y_{t} = \sum_{i=1}^{p} a_i \, y_{t-i} \sum_{i=0}^{q} c_i \, x_{t-i} + \varepsilon_t \quad (12)$$

where ε_t is a non-zero mean error term (white noise) and x_t is a K-dimensional column vector process. The coefficients a_i are (n x n) matrix while c_i are row vectors.

Furthermore, from equation (3.12), the specified BVECM model for the study is given as:

$$\mathrm{GDP}_t = \alpha_0 + \sum_{i=1}^{n} \beta_{2i}\,\mathrm{INFR}_{t-i} \sum_{i=1}^{n} \beta_{3i}\, UER_{t-i} + \varepsilon_{1t} \quad (13)$$

where INF is the inflation rate and UER is the unemployment rate, ε is the error term and t is the time.

Forecasting Assessment

The two performance assessment (measurement) used are Root Mean Square Forecast Error (RMSE) and the Average Logarithmic Predictive Score (ALPS). The result with least values of RMSE shows a robust forecasting assessment while ALPS with large values indicates a better forecasting.

RESULTS AND DISCUSSION

The section of the research work looked at the analysis of macroeconomic variables for the research. The macroeconomic variables are Gross Domestic Product (GDP), inflation Rate and unemployment rate of Nigeria.

Table 1: Descriptive statistics of the Macroeconomic Variables

Descriptive	***Real GDP (=N= billion)***	***Inflation rate (%)***	***Unemployment rate (%)***
Mean	8998.215	17.32172	12.17224
Standard Error	385.9607	1.447993	0.386838
Median	6418.182	11.76655	11.38787
Standard Deviation	4820.647	18.08542	4.831609
Sample Variance	23238639	327.0826	23.34444
Kurtosis	-0.95204	11.63923	-0.43147
Skewness	0.738813	3.314144	0.648098
Range	15825.02	111.1395	18.31409
Minimum	3702.704	-1.88229	5.72591
Maximum	19527.72	109.2572	24.04
Sum	1403721	2702.189	1898.869
Count	156	156	156

The table above shows the descriptive statistics (Mean, Standard Error, Median, Standard Deviation, Sample Variance, Kurtosis, Skewness, Range, Minimum, Maximum, Sum and Count) of the macroeconomic variables used in the analysis.

Time Series plot of the variables

This section shows the time plot for the selected macroeconomic variables data from 1981 to 2019

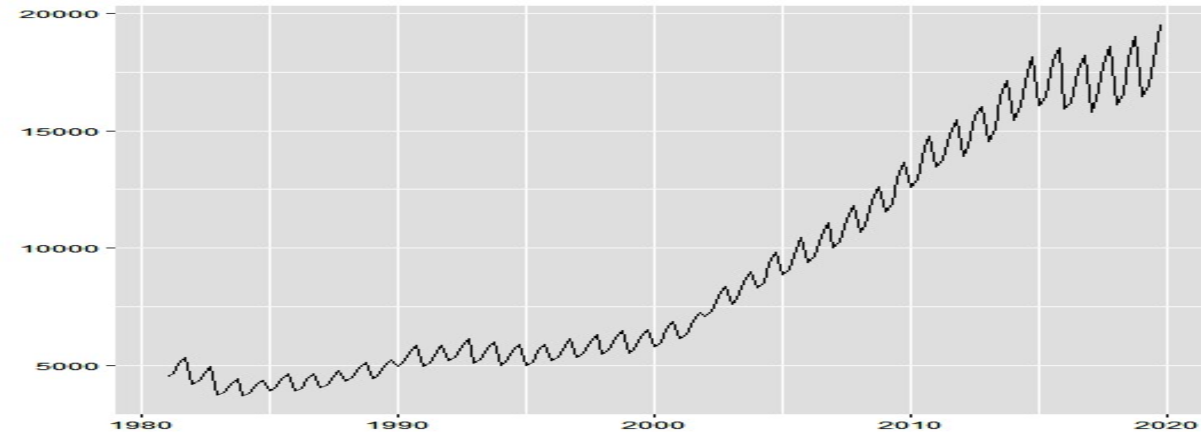

Figure: 1 Time series plot of real GDP

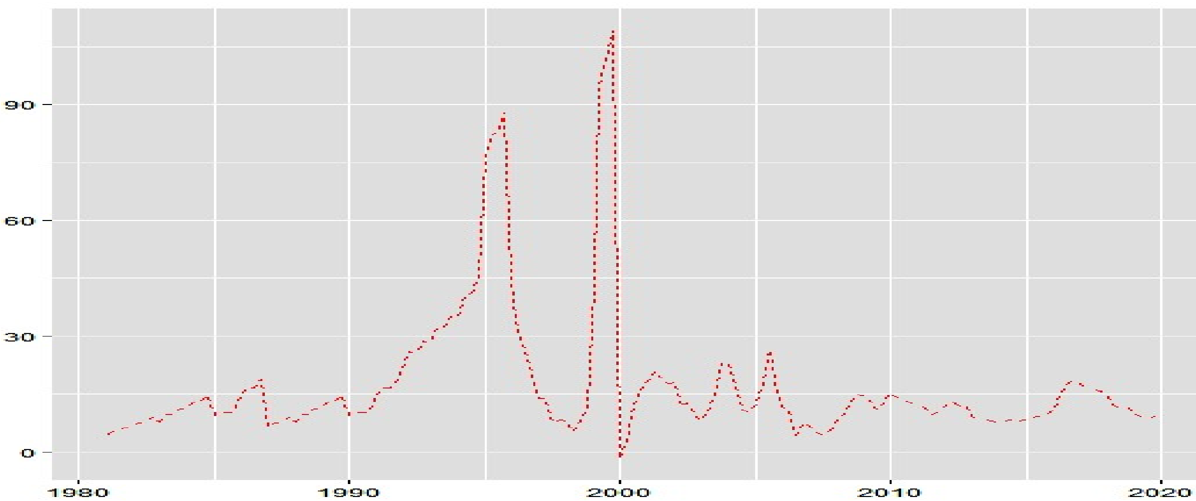

Figure 2: Time series plot of inflation rate

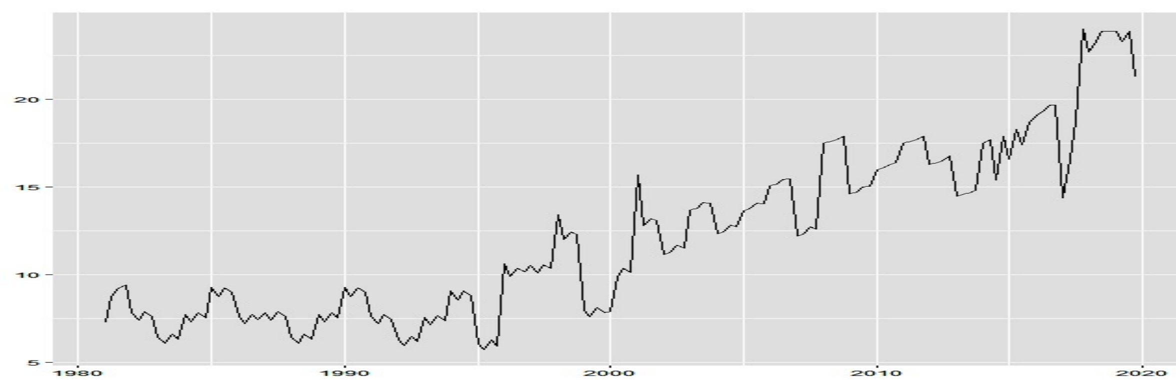

Figure 3: Time series plot of Unemployment rate

This graphs above show the time plot for the selected macroeconomic variables data from 1986 to 2019.

Table2: Johansen Test

Macro-economic Variables	Trace statistic, with linear trend
Real GDP (=N= billion)	0.0206
Inflation rate (%)	0.0169
Unemployment rate (%)	0.0004

Table 2 above shows the Johansen test for the variables under study. The table above shows the trace statistic with linear trend for the variables. From the result, all the macroeconomic variables selected for the study has trace statistic value less than 0.05 and this implies that there exist cointegration relationship among the variables.

Table 3: Co-integration Test: Values of test statistic and critical values of test

Rank	Test	10pct	5pct	1pct
r <= 4	0.06	6.50	8.18	11.65
r <= 3	6.96	15.66	17.95	23.52
r <= 2	20.73	28.71	31.52	37.22
r <= 1	48.77	45.23	48.28	55.43
r = 0	83.64	66.49	70.60	78.87

Table 3 shows the trace test statistic for the hypotheses of $r \leq 4$, $r \leq 3$, $r \leq 2$, $r \leq 1$ and $r \leq 0$. For each of these three tests we have not only the statistic itself (given under the test column) but also the critical values at certain levels of confidence: 10%, 5% and 1% respectively.

The first hypothesis, $r\leq 0$ tests for the presence of cointegration. It is clear that since the test statistic exceeds the 1% level significantly (83.64 > 66.49) that we have strong evidence to reject the null hypothesis of no cointegration. The second test $r \leq 1$ also provides clear evidence (48.77 > 45.23) to reject the null hypothesis since the test statistic exceeds the 1% level significantly. Also, the test statistic of rank 3 is 6.96. the value is less than the critical values of 1%. 5% and 10%. We do not reject the null hypothesis and thereby conclude that there is maximum of 3 cointegration equations. This also reveal that there is a long run relationship among the macroeconomic variables.

Table 4: Estimation of Posterior mean under the three priors under BVECM

Variables	Subjective Prior	Symmetric Prior	Asymmetric Prior
Gdp	1.000	1.000	1.000
Inf	-2001.329	-10681.439	-3755.293
Unemp	-1121.500	-1276.104	-549.533
Constant	-23.459	-116.571	-66.588

Table 5: Estimation of Posterior mean under the three priors under BVAR

Variables	Subjective Prior	Symmetric Prior	Asymmetric Prior
Gdp-1	2.3690	1.2826	1.2246
Infr-1	1.4250	1.0465	1.01231
Uner-1	0.9671	0.9850	0.8590
Constant	-2.5002	-1.1101	-1.0006

From table 4 and 5 above, the posterior mean of the BVECM has the least values. This show that in forecasting macroeconomic variables of a nation using the natural conjugate prior, BVECM is better than BVAR when the long run interrelationship and interdependency is desirable. This is in line with Hapsari el at. (2021).

Table 6: One –to eight quarter s ahead for out of sample using RMSFE

Horizon	Variables	VAR	BVAR	BVECM
h =1	Gdp-1	4.360	1.282	1.224
	Infr-1	1.250	0.835	1.104
	Uner-1	0.425	0.766	0.773
h =2	Gdp-2	3.985	1.106	1.003
	Infr-2	0.776	0.998	0.999
	Uner-2	0.568	0.702	0.741
h =3	Gdp-3	3.762	1.073	0.985
	Infr-3	0.546	0.943	0.921
	Uner-3	0.534	0.970	0.704
h =4	Gdp-4	3.641	1.831	0.963
	Infr-4	1.352	1.002	1.222
	Uner-4	0.160	1.948	1.010
h=5	Gdp-5	3.596	1.904	0.932
	Infr-5	0.536	1.444	1.642
	Uner-5	0.529	2.884	1.564
h=6	Gdp-6	3.552	2.202	0.902
	Infr-6	0.510	1.621	1.862
	Uner-6	2.498	2.741	1.663

h=7	Gdp-7	3.501	2.435	0.845
	Infr-7	0.405	2.863	1.925
	Uner-7	0.452	2.530	1.984
h=8	Gdp-8	3.483	2.648	0.825
	Infr-8	0.433	3.001	2.904
	Uner-8	0.400	2.334	1.982
Average	GDP	3.735	1.8101	**0.9601**
	INFR	0.726	**1.5884**	1.5974
	UNER	0.696	1.8594	**1.2780**

From table 6 above, the values of VAR model under the different horizon are used as the benchmark in selecting the better and robust model in forecasting the main macroeconomic variables of Nigeria economy. The numbers in bold shows the values of the models with the least values of RMSFE. 0.960 of RMSFE under the BVECM shows that it is a better and robust model to forecast GDP against BVAR of 1.8101. Also, from the results, BVAR is adjoined a better model to forecast inflation rate while BVECM model is more robust in Forecasting unemployment in Nigeria. This is in agreement with Chen and Leung (2003) in evaluating the key macroeconomic variables of US where they used the model (BVECM) in forecasting the GDP (Output Growth) and unemployment rate. On the other hand, BVAR has been adjoined by many statistical researchers for forecasting inflation rate as supported by Feldkircher (2017).

Tables 7 and 8 below shows the predictive performance measures for both the BVAR and BVEC models. Table 7 shows the forecast performance for the BVEC and BVAR when the real life data was used, while Table 8 below shows the forecast performance for the BVECM and BVAR model for the simulated data.

Table 7: Forecast Performance for BVAR and BVECM for real life data

BVECM		**BVAR**	
Root Mean Squared Forecast Error (RMSFE)	Average logarithmic predictive scores (ALPS)	Root Mean Squared Forecast Error (RMSFE)	Average logarithmic predictive scores (ALPS)
1.924321e-16	0.9275472	2.330926e-16	0.9875571
1.814332e-16	0.9710888	1.9306e-16	0.9664929
1.84352e-16	0.9858026	1.9330e-16	0.9726844

Table 7 above shows the forecast performance measure for the BVECM and BVAR using the real life data. The two performance measures used are Root Mean Square Forecast Error (RMSFE) and the Average Logarithmic Predictive Score (ALPS). The result shows that the values of the RMSFE for BVECM is smaller compare to that of BVEC produced at all various scale priors. This implies BVECM has a better forecast performance than BVAR.

Table 8: Forecast Performance for BVECM and BVAR for simulation data

	BVECM		**BVAR**	
Prior	Root Mean Squared Forecast Error (RMSFE)	Average logarithmic predictive scores (ALPS)	Root Mean Squared Forecast Error (RMSFE)	Average Logarithmic Predictive Scores (ALPS)
Scale = 0.001	0.2849198	0.89159812	0.5722230	0.7350016
Scale = 0.01	0.2616692	1.20397280	0.4698898	0.7701218
Scale = 0.1	0.1784648	1.60943791	0.2981995	1.0885597

As an extension, the forecasting performance of BVECM were compared with BVAR in simulated data. Table 8 above shows the forecast performance measure for the BVECM and BVAR using the simulated data. The two performance measures used are Root Mean Square Forecast Error (RMSFE) and the Average Logarithmic Predictive Score (ALPS). The result shows that the values of the RMSFE for BVAR model is larger compare to that of BVECM produced at all various scale priors. Also, the ALPS of BVAR is smaller than that of BVECM. This affirmed the real life results in table 8 and support the assertion that BVECM performs better than BVAR when the explicitly long-run relationship if forecasting of macroeconomic variable is required.

Table 9: Forecast Performance for BVECM and BVAR

	BVAR		BVECM	
Prior	**RMSFE**	**Accuracy**	**RMSFE**	**Accuracy**
Subjective	1.924321e-16	0.995732	2.330926e-16	0.997402
Symmetric	1.914332e-16	0.995888	1.5306e-16	0.995810
Asymmetric	1.904352e-16	0.993743	1.0330e-16	0.995960

Table 9 above shows the accuracy of BVAR and BECM. It is evident that BVECM model has a better forecasting accuracy than BVAR model. The forecasting accuracy of BVAR model under the different priors are (99.6%, 99.6% and 99.4%) against that of BVECM which is (99.7%, 99.7% and 99.6%). This reveals that BVECM has a better forecasting accuracy than BVAR.

CONCLUSION

The study introduces the paradigm shift from the conventional BVAR model of forecasting and estimating macroeconomic variables of a nation. This shift is occasioned by the limitations associated with BVAR such as BVAR misspecification and its inability to elucidate information relating to the integration and cointegration among the macroeconomic variables. Also, the predictive accuracy of BVAR model has some blemish owing to the fact that it lacks the estimation capacity to elucidate vital information about the co-movement among macroeconomic variables of a developing economy like Nigeria. It also produces more accurate out-of-sample forecast than BVAR and predict the direction of change in the chosen macroeconomic variables. Therefore, in forecasting and evaluating main macroeconomic variables of a nation, BVECM should be considered because it possesses a better predictive ability than the BVAR model since it predicts the direction of change in the chosen macroeconomic variables. Also, it also gives a vital information about the interdependency of the macroeconomic variables.

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