STUDY OF STORAGE AND RETRIEVAL OF SINGLE-PHOTON QUANTUM STATE IN A COLLECTIVE ATOMIC EXCITATIONS

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ABSTRACT

In this article, the probe pulse (that is, of single-photon quantum states) which can be stored in long live cavity dark state is investigated. The transfer of the single photon state to the collective cavity dark state can be accomplished with nearly a hundred percent efficiently by the method of intracavity EIT and assumption of quantum impedance matching condition whereby the classical driving field is adiabatically switched off while the probe laser is inside the dense ensemble of atoms and cos θ is optimized for any input pulse respectively is clearly described. It is noted that, this method of using an optically-dense many-atom medium is far superior and efficient than the technique with single atoms processed in the strong coupling system.

Key Words: Quantum Impedance Matching, Coupling of cavity-dark state, Free-field modes, Adiabatic Following, Electromagnetic induced Transparency (EIT), Intracavity Electromagnetic Induced Transparency

INTRODUCTION

The coherent control of quantum systems has been given much attention and experimental effort. Its applications range from the possibility of studying fundamental aspects of quantum Physics to quantum information processing. The practical implementation of quantum processing protocols requires coherent manipulation of a large number of coupled quantum systems which is an extremely difficult task. A wide variety of physical systems and phenomena have been proposed as quantum systems. Some of them are trapped ions, atoms in optical lattices, cavity quantum electrodynamics (Cavity QED), as well as the three-level atom.

Coherence effects in three-level atom system bring about two interesting but related phenomena: Coherent Population Trapping (CPT) and Electromagnetically Induced Transparency (EIT), hence these concepts are closely related and one precedes the other (Khan *et al*., 2017). Knutson, (2020) discovered that the fluorescence from a gas of sodium atoms vanishes when the splitting of the hyperfine levels of the atoms matches the mode spacing of the applied multimode laser. The discovery of CPT has led to the realization that an otherwise opaque medium could be rendered transparent by the application of two coherent laser fields that allows for optical pumping between allowed transitions for which all the population (atoms and their electrons) decay into dark non absorbing state (dark state). This later on led to the realization of the concept of Electromagnetically Induced Transparency (EIT).

The initial investigational demonstration of EIT was reported in 1991 by (Boller *et al*., 1991) using a gas of strontium atoms. Subsequent experiments (Kasapi *et al*., 1995, Schmidt *et al*., 1996, Kash *et al*., 1999; Shiet *et al*., 2015; Khoa *et al*., 2017) focused not only on demonstrating transparency of the medium, but also on measuring the associated reduction of the speed of light. Zafar and his co-workers were able to use EIT to measure light speeds as low as 17 m/s (Zafar and Salim, 2015). Shortly after, it became apparent that EIT had potential applications in the field of quantum information processing as it was demonstrated that the laser pulses could not only be slowed down, but in fact brought to a complete stop in the medium. It thus became possible to use the medium as a quantum memory, transferring the quantum state of free-field photons to the ensemble of gas phase atoms and retrieval of these states.

Succeeding experiments demonstrates that EIT can be used as a control tool to compress and spatially stop the probe pulse and regenerate this pulse at a later time in a cloud of cold sodium atoms (Liu *et al*., 2001). Light consist of photons, so photons can be made ideal carriers of quantum information because: they travel at the speeds of 300,000km/s, they are robust and readily available. On the other hand atoms present reliable and long-lived storage and processing units. Therefore, the challenge is to develop a technique for coherent transfer of quantum information carried by light to atoms and vice versa. In other words, it is necessary to have a quantum memory that is capable of ”storing” and ”releasing” quantum states on the level of individual photon qubits. Such a device needs to have a stringent requirement of coherence in order to achieve an-unidirectional transfer (from field to atoms or vice versa)

Therefore this article focuses on schemes for trapping, storage, and retrieval of single-photon quantum states of the probe pulse within an optically dense coherently driven electromagnetically-induced transparent medium inside an optical resonator by the method of intracavity EIT. This technique deals with the manipulation of the properties of a cavity filled with three-level Λ-type atoms by an external classical field. The single-photon quantum states will be considered for storage in the dark state formed by the two coherent fields interacting with an optically dense medium simultaneously. Other techniques for storage and retrieval of light utilizes dark-state polaritons with EIT technique (Fleischhauer *et al*., 2000). This form-stable (dark-state polariton) coupled excitation of light and matter has been used in three-level Λ atom to establish quantum memories for light. In this paper, the storage and retrieval of the probe pulse for which these pulses are multiplexed and injected into the cavity system at a regular period is studied. This time-entangled pulses are considered for storage under adiabatic photon transfer schemes under dynamical quantum impedance matching conditions. The concept of stopped light resulting from EIT will be utilized in the storage and retrieval of quantum states of photons

The paper also emphases on re-deriving and generalizing the equations governing the storage and retrieval of single-photon quantum states in the quantum states of collective atomic excitations in an electromagnetically-induced transparent medium recently established by (Distante *et al*., 2017). After establishing these governing relations, the storage and retrieval of a single high intensity pulse described mathematically by the hyperbolic secant pulse. This pulse is injected into this optically dense coherently driven electromagnetically induced transparent medium inside a single-mode cavity by the technique of intracavity electromagnetically induced transparency (EIT) is investigated (Fleischhauer *et al*., 2000). For this reason, a single-photon wave packets is referred to as photon guns (Vallone *et al*., 2016). The underlying physical mechanism of adiabatic photon storage and subsequent retrieval is based on intracavity EIT by which properties of a cavity filled with three-level Λ-type atoms are manipulated by an external control field. The control field is vital in the loading and unloading process of cavity filled electromagnetically induced-transparent medium and should always be treated in an adiabatic fashion.

The Λ-type three-level atom and dark state

Considering an optically dense ensemble of N identical three-level atoms confined within an optical cavity. Of the two dipole-allowed transitions one is coupled to the cavity mode with a coupling constant g, while the other optical transition is driven by a field with Rabi frequency $\Omega(t)$ as depicted in Fig 1. The interaction Hamiltonian

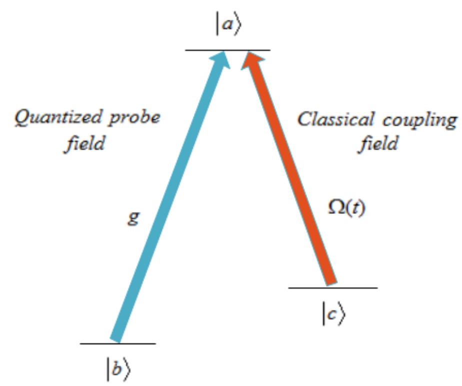

Figure 1: The three-level atom with a quantized field and classical field. Of this system can be expressed as the sum of the two dipole-allowed transitions in the Λ-type three level atom system transition pathways given by:

$$H_{\text{int}} = H_{a-b} + H_{a-c}, \qquad \text{Eq. (1)}$$

The meta-stable states ($|c\rangle$ and $|b\rangle$) interact with the excited state through two different transitions, the probe laser (quantized field) and the coupling laser (classical field) respectively and initially, all the population is in the state $|b\rangle$. The interaction Hamiltonian resulting from eq.(1), was defined by (Fleischhaeur *et al*., 2000) as follows:

$$H = \hbar g \sum_{i=1}^{N} \hat{a}\hat{\sigma}_{ab}^{i} + \hbar\Omega(t) e^{-ivt} \sum_{i=1}^{N} \sigma_{ac}^{i} + h.c.$$

Eq. (2)

Here, $\sigma_{\mu\nu}^{i} = |\mu\rangle_{ii}\langle\nu|$ is the flip operator of the *i*th atom between states $|\mu\rangle$ and $|\nu\rangle$, where μ, ν = *a*, *b*, *c*. *g* is the coupling constant between the atoms and the field modes (vacuum Rabi frequency), *h.c* stands for the Hermitian conjugate, and $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and creation operators respectively. For simplicity,we assume that *g* is equal for all atoms. The first term of the interaction Hamiltonian describes the process in which atoms are excited from the lower energy state $|b\rangle$ to the upper state $|a\rangle$ and single photon modes are annihilated. The second term of this Hamiltoniandescribestheprocessinwhichatomsaretakenfromtheexcited state $|a\rangle$ to the lower state $|b\rangle$ and single photon modes are created. The terms in the h.c. describes the reverse process respectively of these two terms.

We define a collective atomic operator (Welter *et al*., 2018) by the sum of flip operators of every atom,

$$\sigma_{ab} = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \sigma_{ab}^{i}, \quad \sigma_{ac} = \sum_{i=1}^{N} \sigma_{ac}^{i} \qquad \text{Eq. (3)}$$

With N the number of atoms and assuming that only symmetric collective states are involved in the process. With these new operators, the part of the total Hamiltonian describing the interaction between photons and atoms will take the form:

$$H_{\text{int}} = \hbar g \sqrt{N} \hat{a} \sigma_{ab} + \hbar\Omega(t) e^{-ivt} \sigma_{ac} + h.c.$$

Eq. (4)

where the interaction strength between photons and the collective atomic excitations is assumed $\sqrt{N}$ times larger than the interaction between the light and a single atom, such that only certain specific collective states of the atomic excitations are allowed. Of special interest are superposition states of light and collective states of matter that do not interact with the optical fields, namely the so-called dark state. More explicitly the dark state is obtained under the condition of two-photon resonance from the interaction Hamiltonian, when the energy difference between the metastable states equals the energy difference per photon of the two fields. The resulting eigenstate takes the following form:

$$|D\rangle = \frac{\Omega|b\rangle - g\sqrt{N}|c\rangle}{\sqrt{\Omega^2 + g^2 N}} = \cos\theta(t)|b\rangle + \sin\theta(t)|c\rangle$$

Eq. (5)

where $\cos\theta(t) = \frac{\Omega}{\sqrt{\Omega^2 + g^2 N}}$ and $\sin\theta(t) = \frac{g\sqrt{N}}{\sqrt{\Omega^2 + g^2 N}}$, with $\theta(t) = \arctan\left(\frac{g\sqrt{N}}{\Omega}\right)$ the mixing angle. The bright-eigenstate also corresponding to the same interaction is given by:

$$|B\rangle = \frac{g\sqrt{N}|b\rangle - \Omega|c,0\rangle}{\sqrt{\Omega^2 + g^2 N}} = \sin\theta(t)|b\rangle + \cos\theta(t)|c\rangle$$

Eq. (6)

The adiabatic single-photon state transfer into the dark state is still obtained at the two-photon resonance and the dark state corresponding to a single excitation in one of the metastable state (see fig. 1) is given by:

$$|D,1\rangle = -i\frac{\Omega|b,1\rangle - g\sqrt{N}|c,0\rangle}{\sqrt{\Omega^2+g^2N}} = -i\cos\theta(t)|b,1\rangle + i\sin\theta(t)|c,0\rangle$$

Eq. (7)

Where the phase factor $-i$ is introduced and has no effect on the population in the dark state, the photonic number state inside the cavity mode is denoted by $|n\rangle$ and the mixing angle is defined as above.

It is noted note from eq. (7) is analogous to eq. (5) and therefore all concepts applied in the case of the two classical fields will be applied here. It is convenient to note that the state $|D\rangle$ decouples from both classical and quantum fields due to interference. This is evident because the dark state $|D,1\rangle$ has no component of $|a,0\rangle$ but rather a linear superposition of the meta-stables states $|b\rangle$ and $|c\rangle$ Also, the action on the dark state by the interaction Hamiltonian eq. (9) shows the decoupling of the dark state from the two fields,

$$H|D,1\rangle = 0 \qquad (8)$$

This shows that all population is essentially trapped in the dark state with one cavity photon. A particular case of interest is when $g\sqrt{N} \gg \Omega$. The state $|D,1\rangle$ reduces to:

$$|D,1\rangle \approx \frac{ig\sqrt{N}}{\sqrt{\Omega^2+g^2N}}|c,0\rangle \qquad (9)$$

The state $|D,1\rangle$ corresponds nearly identical to the state $|c,0\rangle\left(i.e.\ |D,1\rangle \approx |c,0\rangle\right)$. In this case, the single excitation is basically shared amongst the atoms. After constructing the dark state that holds up all the population, we will then look at the principle that would be used to achieve the mechanism of storage and retrieval.

The principle of intracavity EIT proposed by Fleischhauer *et al*., (2000) will now be discussed. To have a complete description of the interaction, we take into account dissipation and decay into the analysis. With respect to dissipation in this system, three important mechanisms must be analyzed:

1. The dark state $|D,1\rangle$ is not susceptible to decay from the excited atomic levels (i.e. state $|a\rangle$) since the $|D,1\rangle$ has no component of $|a\rangle$.
2. The dark state is susceptible to decay of its constituents (lower level coherence between $|b\rangle$ and $|c\rangle$). The decay γ_{bc} from $|D,1\rangle$ sets the ultimate (maximum) upper limit on the life-time of the dark state $|D,1\rangle$.
3. The effect of the finite $Q-$ value of the cavity brings about the cavity decay. The bare cavity has a decay rate given by γ. This cavity decay rate γ will lead to the decay of the dark state. The only part of the dark state that can decay is the one that contains a single photon. i.e the part proportional to $|b,1\rangle$. The probability of being in state $|b,1\rangle$ is given by:

$$P(t) = \left|\langle b,1|D,1\rangle\right|^2 \qquad \text{Eq. (10)}$$

and this leads to

$$P(t) = \left|-i\cos\theta(t)\right|^{2} = \cos^{2}\theta(t) \quad \text{Eq. (11)}$$

Therefore, the decay of the dark state due to the cavity is given by:

$$\frac{\gamma D}{2} = \frac{\gamma}{2}\cos^{2}\theta(t) \qquad \text{Eq. (12)}$$

where the factor $\frac{1}{2}$ is just a convention in the definition of decay and thus the cavity decay rate is $\frac{\gamma}{2}$.

Furthermore, if $\cos^{2}\theta(t)$ □ 1, this corresponds to the case $g\sqrt{N}$ □ Ω, the effect of the cavity decay rate is considerately reduced in the limit Eq. (7) is given by:

$$\left|D,1\right\rangle = -i\frac{g\sqrt{N}}{\sqrt{\Omega^{2}+g^{2}N}}\left[\frac{\Omega}{g\sqrt{N}}\left|b,1\right\rangle - \left|c,0\right\rangle\right]$$

Eq. (13)

It is apparent that $\left|D,1\right\rangle$, contains only small (□ $\frac{\Omega}{g\sqrt{N}}$) component of the single photon state $\left|b,1\right\rangle$, this brings about an increase in the lifetime of the combined atom-cavity system and it's an extremely important feature of intracavity EIT. Intracavity EIT has one more important feature which is obtained by varying the mixing angle θ (t) (i.e. by adiabatically switching off and on the Rabi frequency of the classical driving field Ω (t)), the coupling of dark state to its environment is attained. This is entirely related to the concept of stopped light whereby one can adiabatically switch off the Rabi frequency for the storage process and later on switch it back on for the retrieval process.

Single-photon Excitation by Adiabatic Coupling of Cavity-dark state to free-field modes

Consider the coupling of the cavity-dark state to free-fields modes inside the cavity. The optically dense ensemble of atoms is placed inside the Fabry-Pérot cavity (resonator). The z-axis is parallel to the propagation of the input and outgoing modes. The point $z = 0$ characterizes the position of the partially transmitting input mirror of the cavity. The other mirror of the cavity is assumed to be totally reflecting. The configuration of the dense ensemble of atoms in the Fabry-Pérot cavity is depicted in Fig. 2.

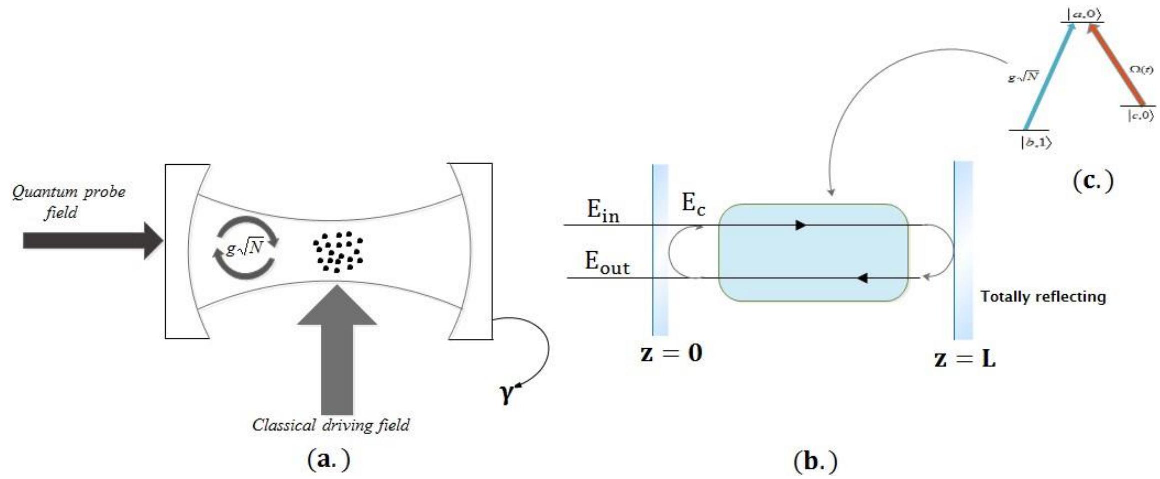

Figure 2: (a.) Cavity set up with N atoms interacting with the cavity mode and a classical control field. γ is the empty cavity decay rate. (b.) Simplified optical cavity with an optically dense ensemble of atoms .E_c, E_{in}, E_{out} are circulating, input, and output components of the field. (c.) shows that the atoms in the optical medium are described by 3-level atom.

In order to construct the Hamiltonian governing the input-output process, we need to first of all consider the field inside and outside the cavity. As stated in chapter one, the field inside the cavity with a single mode is governed by the Hamiltonian given by

$$H = \hbar\omega_c \hat{a}^\dagger \hat{a}, \qquad \text{Eq. (14)}$$

where ω_c is the angular frequency of the field in the cavity and $\hat{a}^\dagger$, $\hat{a}$ are the creation and annihilation operators of the field within the cavity respectively. The field outside this cavity has many modes and we denote the modes by 'k'. Therefore, the Hamiltonian governing the field outside this cavity is given by:

$$H = \hbar\omega_f \hat{b}_k^\dagger \hat{b}_k \qquad \text{Eq. (15)}$$

where ω_f is the angular frequency of the free-space field outside the cavity and $\hat{b}_k^\dagger$, $\hat{b}_k$ are the creation and annihilation operators of the field outside the cavity with mode k.

In order to model the coupling of the free-space modes to the selected cavity mode, a continuum of free-space-modes with field operators $\hat{b}_k$ couples to selected cavity mode with a coupling constant κ. For simplicity, we consider that the coupling constant is the same for all the relevant modes. The interaction Hamiltonian describing the coupling of the cavity and this free field mode is of the form:

$$H_{cav-free} = \hbar k \sum_k \hat{a}^\dagger \hat{b}_k + h.c. \qquad \text{Eq. (16)}$$

where $h.c$ stands for Hermitian conjugate, $\hat{a}^\dagger$ is the creation operator and $\hat{b}_k$ is the annihilation operator of the free-field with mode k. eq.(16) is a reasonable ansatz because we assume that the single cavity mode is linearly coupled to the outside modes and in the most simple case, the coupling is the same for all 'k'-modes. The linear coupling guarantees that if double the field strength outside, one can also find the field strength to be doubled inside. The first term in eq. (16) describes a process in which a photon in the outside world mode 'k' will be annihilated and at the same time a photon inside the cavity is created while the Hermitian conjugate, describes the opposite process where a cavity photon is annihilated and an outside photon in (an arbitrary) mode 'k' is created.

Let us examine the representation of the input single-photon quantum states. Since the number states $|n\rangle$ form a complete set, we may expand the input wave packet $|\psi_{in}(t)\rangle$ according to $|\psi_{in}(t)\rangle = \sum_{n=0}^{\infty} \xi_n(t)|n\rangle$. Also, our input pulses are assumed of single-state photons with various modes k so they can be represented in a general single-photon state as:

$$|\psi_{in}\rangle = \sum_{n=0} \xi_k^{in}(t)|1_k\rangle \qquad \text{Eq. (17)}$$

where $\xi_k^{in}(t) = \xi_k^{in}(t_0)e^{-i\omega_k(t-t_0)}$ and $|1_k\rangle$ is a bosonic Fock state and stands for $|0,\ldots,1_k,\ldots,0\rangle$ and $\sum_k |\xi_k^{in}|^2 = 1$. In what follows, the field going to be described by an envelope wave function $\Phi_{in}(z,t)$ defined by:

$$\Phi_{in}(z,t) = \sum \langle 0_k | \hat{b}_k e^{ikz} | \psi_{in} \rangle \qquad \text{Eq. (18)}$$

Here we change to the continuum limit by performing the following transformation: $\xi_k(t) \to \xi(\omega_k, t)$ and $\sum_k \left(\frac{L}{2\pi}\right)\int dk$ where L is the quantization length. This transform eq. (18) into:

$$\Phi_{in}(z,t) = \frac{L}{2\pi c}\int d\omega_k \xi^{in}(\omega_k, t)e^{ikz} \qquad \text{Eq. (19)}$$

Equation (19) admits a soliton from the nonlinear Schrödinger equation. The normalization condition $\frac{L}{2\pi c}\int d\omega_k |\xi^{in}(\omega_k,t)|^2 = 1$ of Fourier coefficient implies the normalization of the input wave function.

$$\int \frac{dz}{L}|\Phi_{in}(z,t)|^2 = 1 \qquad \text{Eq. (20)}$$

The Input-Output Problem

If the input single-photon wave-packet interacts with the combined systems of cavity-mode and the atoms, the general state can be written in the state given by:

$$|\psi(t)\rangle = b(t)|b,1,0_k\rangle + a(t)|a,0,0_k\rangle + c(t)|c,0,0_k\rangle + \sum \xi_k(t)|b,0,1_k\rangle \qquad (21)$$

where the state $|b,1,0_k\rangle$ denotes the state corresponding to the atomic system in the collective state $|b\rangle$, the cavity mode in single-photon state $|1\rangle$ and there are no photons in the outside modes $|0_k\rangle$ combining eqs.(4) and (6) the total interaction in the atom-cavity together with the free-field modes is given by:

$$H = \hbar g\sqrt{N}\hat{a}\sigma_{ab} + \hbar\Omega(t)e^{-ivt}\sigma_{ac} + \hbar k\sum_k \hat{a}^{\dagger}\hat{b}_k + h.c. \qquad \text{Eq. (22)}$$

We lay down the relation governing the

input-output process by taking into account the two-photon resonance condition. This assumption requires the bare frequency of the cavity mode to coincide with the $a-b$ transition of the atom as well as the carrier frequency of the input wave-packet i.e $v_c = \omega_{ab} \equiv \omega_a - \omega_b = \omega_0$ and also the classical driving field should be in resonance with the $a-c$ transition, i.e $v = \omega_{ac}$. The equations of motion are obtained by solving the Schrodinger equation in the interaction diagram given by:

$$i\hbar \frac{d}{dt}|\psi\rangle = H|\psi\rangle \qquad \text{Eq. (23)}$$

where $|\psi\rangle$ is a state vector of the system. The relaxations relating to the spontaneous emission and dephasing of the system is added phenomenologically thought it can be derived quantum mechanically when one considers the joint density matrix of the atom and all modes of the vacuum radiation field. In general, considering the motion of resonant atomic systems, the density matrix equations can be adopted. Nevertheless, as demonstrated in (Lukin et al., 1998) for EIT – like coherent atomic systems, the density matrix equations can be replaced by the probability amplitude equations without any difference. Evaluating eq.(28) the time evolution of the amplitudes are given by

:

$$\dot{a}(t) = -\frac{\gamma}{2}a(t) - ig\sqrt{N}b(t) - i\Omega c(t) \qquad \text{Eq. (24)}$$

$$\dot{b}(t) = -ig\sqrt{N}a(t) - ik\sum_k \xi_k(t) \qquad \text{Eq. (25)}$$

$$\dot{c}(t) = -i\Omega a(t) \qquad \text{Eq. (26)}$$

$$\dot{\xi}_k(t) = -i\Delta_k \xi_k(t) - ikb(t) \qquad \text{Eq. (27)}$$

where $\Delta_k = \omega_k - \omega_0$ is the detuning of the free-field modes from the cavity resonance and $\omega_0 = v_c = \omega_{ab}$. To describe the adiabatic transfer process, it commence by incorporating eq.(6) and (7) into eqs.(24)-(27) and obtain the corresponding probability amplitude equations given by:

$$\dot{a}(t) = -\frac{\gamma_a}{2}a(t) - i\Omega_0 B(t) \qquad \text{Eq. (28)}$$

$$\dot{B}(t) = -i\dot{\theta}(t)D(t) - i\Omega_0 a(t) - ik\sin\theta(t)\sum_k \xi_k(t) \qquad \text{Eq. (29)}$$

$$\dot{D}(t) = i\dot{\theta}(t)B(t) + k\cos\theta(t)\sum_k \xi_k(t) \qquad \text{Eq. (30)}$$

$$\dot{\xi}_k(t) = -i\Delta_k \xi_k(t) - ik\sin\theta(t)B(t) - k\cos\theta(t)D(t) \qquad \text{Eq, (31)}$$

Terms proportional to $\dot{\theta}(t)$ describe non-adiabatic coupling between the bright and dark state. We cab adiabatically eliminate the excited state and bright state amplitude and disregard non-adiabatic corrections. This elimination leads to the following equations:

$$\dot{D}(t) = k\cos\theta(t)\sum_k \xi_k(t) \qquad \text{Eq, (32)}$$

$$\dot{\xi}_k(t) = -i\Delta_k \xi_k(t) - k\cos\theta(t)D(t) \qquad \text{Eq, (33)}$$

Solving the above two equations simultaneously and obtain:

$$\frac{d}{dt}\left(\left|D(t)\right|^2 + \sum_k \left|\xi_k\right|^2\right) = 0 \qquad \text{Eq, (34)}$$

Equation (34) shows that, the total probability of finding the system in a free-field single-photon or in the cavity dark state is conserved. Therefore, under adiabatic conditions, there is only an exchange of probability between free-field state and the cavity dark state. After the input process, we now determine the form of the output field. Changing to the continuum limit by performing the following transformation: $\xi_k(t) \rightarrow \xi(\omega_k, t)$. Eq.(3.33) becomes:

$$\dot{\xi}(\omega_k, t) = -i\Delta_k \xi(\omega_k, t) - k\cos\theta(t)D(t). \qquad \text{Eq, (35)}$$

Integrating Eq. (35), leads to:

$$\xi(\omega_k, t) = \xi^{in}(\omega_k, t_0)e^{-\Delta_k(t-t_0)} - k\int_{t_0}^{t} d\tau \cos\theta(\tau)D(\tau)e^{-i\Delta_k(t-\tau)} \qquad \text{Eq, (36)}$$

Taking Eq. (35) into the continuum light and obtain:

$$\dot{D}(t) = \frac{kL}{2\pi c}\cos\theta(t)\int d\omega_k \xi^{in}(\omega_k, t_0)e^{-i\Delta_k(t-t_0)} \qquad \text{Eq. (37)}$$

and substituting Eq. (36) into (37) leads to:

$$\begin{aligned}\dot{D}(t) = &\frac{kL}{2\pi c}\cos\theta(t)\int d\omega_k \xi(\omega_k, t) \\ &- k^2\cos\theta(t)\int_{t_0}^{t} d\tau \cos(\tau) D(\tau)\frac{L}{2\pi c}\int d\omega_k e^{-i\Delta_k(t-\tau)}\end{aligned} \qquad \text{Eq, (38)}$$

In the first term, the identified wave function of input photon at z = 0. In the second term, using the Markov-limit $\int d\omega_k e^{-i\Delta_k(t-\tau)} \rightarrow 2\pi\delta(t-\tau)$, and this reduces to:

$$\dot{D}(t) = \frac{kL}{2\pi c}\cos\theta(t)\int d\omega_k \xi^{in}(\omega_k, t_0)e^{-i\Delta_k(t-t_0)} - \frac{k^2 L}{c}\cos\theta(t)\int_{t_0}^{t} d\tau \cos\theta(\tau)D(\tau)\delta(t-\tau) \qquad \text{Eq, (39)}$$

Utilizing the Dirac delta property $\int_{-\infty}^{+\infty} d\tau\delta(t-\tau) = f(t)$, the first ODE given by:

$$\dot{D}(t) = \sqrt{\gamma \tfrac{c}{L}} \cos\theta(t)\Phi(t)_{in}(0,t) - \tfrac{\gamma}{2}\cos^2\theta(t)D(t) \qquad \text{Eq, (40)}$$

Where the empty-cavity decay rate $\gamma = \frac{k^2 L}{c}$ is introduced and set:

$$\Phi_{in}(0,t) = \frac{L}{2\pi c}\int d\omega_k \xi^{in}(\omega_k, t_0) e^{-i\Delta_k(t-t_0)}, \qquad \text{Eq, (41)}$$

In order to solve eq.(40) for D(t), we can simplify the above equation by posing $p(t) = \frac{\gamma}{2}\cos^2\theta(t)$ and $f(t) = \sqrt{\gamma \frac{c}{L}} \cos\theta(t)\Phi_{in}(0,t)$, then:

$$\dot{D} + p(t)D(t) = f(t) \qquad \text{Eq, (42)}$$

Using the Green function technique for first order ordinary differential equation (ODE), we have obtain the solution of Eq. (42) given by:

$$D(t) = \int_{t_0}^{t} e^{-\int_{\tau}^{t} p(\tau')d\tau'} f(\tau)d\tau. \qquad \text{Eq, (43)}$$

Substituting the expressions of $p(t)$ and $f(t)$, taking into account the time t_0 sufficiently before any excitation of the above equation is given by:

$$D(t)\sqrt{\gamma \tfrac{c}{L}}\int_{t_0}^{t} \cos\theta(\tau)\Phi_{in}(0,\tau) e^{-\frac{\gamma}{2}\int_{t_0}^{t}\cos^2\theta(\tau')d\tau'} d\tau. \qquad \text{Eq, (44)}$$

The above equation shows that, there is a linear relation between the dark state D(t) and the input field as it enters the cavity system at z = 0. One observe that, $\cos\theta(t)$ has to be small enough to enhance the transfer of single-photon states from the applied input field. A direct substitution of eq.(44) into (36) yields the input-output relation given by:

$$\Phi_{out}(0,t) = \Phi_{in}(0,t) - \gamma\cos(t)\int_{t_0}^{t} d\tau\cos\theta(\tau)\Phi_{in}(0,\tau) e^{-\frac{\gamma}{2}\int_{\tau}^{t}\cos^2\theta(\tau')d\tau'} \qquad \text{Eq, (45)}$$

In view of the condition for adiabatic elimination of the bright state amplitude. By substituting eq.(45) into (29-32) and taking into account the Markov limit, we find that, the adiabatic following occurs when:

$$\Omega_0^2 \gg \gamma\gamma_a, \quad \Omega_0^2 \gg \frac{\gamma_a}{T}, \quad \Omega_0^2 \gg \sqrt{\frac{\gamma_a}{T}\gamma_a}\,. \qquad \text{Eq, (46)}$$

It is worth nothing that, these conditions also ensure the spontaneous Raman scattering in order that the cavity mode are negligible. Since the characteristic input-pulse length and the characteristic times T have to be larger or equal to the cavity decay time γ^{-1}. The first condition is the precise one.

It is relevant to take note that, in order to ensure adiabaticity it is sufficient that:

$$g^2 N \gg \gamma\gamma_a \qquad \text{Eq, (47)}$$

This condition opposes that corresponding to an adiabatic transfer with a single atom. In case of a single-atom, the strong-coupling regime corresponding (at least) to $g^2 \geq \gamma\gamma_a$.

Looking at the implications of eqs. (53) and (54) if $\cos\theta$ is constant in time, the atom would just cause a change of cavity decay rate, according to $\gamma \rightarrow \gamma\cos^2\theta(t)$. Therefore, by increasing the atom density and equally decreasing $\cos\theta$, the effective life time of cavity mode can be increased.

The method which permits one to capture and subsequently release a single-photon state of light field is now described. To achieve this objective, a techniques of adiabatic transfers is utilized (Ottaviani *et al*., 2006). To give reason to the analysis carried out below, we note that, the state $|D,1\rangle$ eq. (7) couples to the free-field light modes only by means of a mixture of state $|b,1\rangle$ From eq. (32), it is observed that, the dark states coupling to the free field light modes depend on $\cos\theta(t)$. To store the photon state, we first accumulate the field in a cavity mode and then, adiabatically switching off the driving field $\Omega(t)$, an initial free-space wave packet can be stored in a long-lived atom-like dark state. The stored free space wave packet can be released by simply adiabatically turning on the Rabi frequency of the driving field. The storing and the release process would now be discussed thoroughly.

Optimization of Input: Quantum Impedance Matching

The quantum Impedance Matching would be in principle, optimizing the time dependence of $\cos\theta(t)$ such that, the dark-state amplitude will asymptotically get close to unity. This can occur for a bandwidth of the incoming wave which is less or at most equal to the bare cavity bandwidth. i.e, for a wave packet which is longer than the bare-cavity decay time γ^{-1}. Furthermore, the time when the adiabatic transfer starts must coincide with the arrival time of the photon wave packet.

In order to achieve a maximum transfer of free-field photons into cavity photons, the outgoing field components should be minimized. This can be accomplished by using the destructive interference between the directly reflected and circulatory components of the input field. The condition for destructive interference for the directly reflected and circulatory field can be obtained by time derivative of eq. (3.45) and setting $\Phi_{out} = \dot{\Phi}_{out} = 0$ this leads to:

$$-\frac{d}{dt}\ln\cos\theta(t)+\frac{d}{dt}\ln\Phi_{in}(t)=\frac{\gamma}{2}\cos^2\theta(t)$$

Eq, (48)

This equation has a physical interpretation; the first term on the LHS determines the amplitude loss rate of the photon field inside the cavity. The term on the RHS is the effective amplitude decay rate due to the cavity losses (see eq.12). Therefore, if Φ_{in} is constant, eq. (48) constitutes what in classical system is known as impedance matching condition (Siegmann *et al*., 1986). There is need for modification of classical impedance matching conditions when the input field is time dependent, as the circulating field experiences a slightly changed input field after one cavity-round trip, which leads to the second term on the LHS of eq. (48). A solution of eq. (48) is given by:

$$\cos(t)=\frac{1}{\sqrt{\gamma}}\frac{\Phi_{in}(t)}{\sqrt{\int_{t_0}^{t}\Phi_{in}^2(t')\,dt'}}$$

Eq, (49)

In general, for any input field Φ_{in} the mixing angle resulting from eq. (48) gives rise to the Rabi frequency of classical driving by noting that $\Omega(t)=g\sqrt{N}\cot\theta(t)$ leading to:

$$\Omega(t)=g\sqrt{N}\frac{\Phi_{in}(t)}{\sqrt{\gamma\int_{t_0}^{t}\Phi_{in}^2(t')\,dt'-\Phi_{in}^2(t)}}$$

Eq, (50)

Therefore, since the adiabatic photon transfer occurs under the condition of dynamical impedance matching the dark state corresponding to this condition simplifies to the following relation by substituting eq. (49) into (44) as:

$$D(t)=\sqrt{\frac{c}{L}}\frac{1}{\sqrt{\int_{t_0}^{t}\Phi_{in}^2(t')\,dt'}}\int_{t_0}^{t}\Phi_{in}^2(\tau)\,d\tau$$

Eq, (51)

From the above relation of the dark state, one finds that the transfer of free-field photons into the dark state is proportional to the coupling of outside modes of the single mode cavity and the dark state is inversely proportional to the square root in accordance with $\sqrt{c/L} = k/\sqrt{\gamma}$.

Single Input Pulse

The remarkable performance of the adiabatic transfer mechanism under conditions of quantum impedance matching can simply be demonstrated in this sub heading. Since eq.(48) relies explicitly on the shape, consider a normalized hyperbolic secant input pulse which is a single soliton pulse carrying single-photon quantum states given by:

$$\Phi_{in}(z=0,t)=\Phi_{in}(t)=\sqrt{\frac{L}{cT}}\sec h\left(\frac{2t}{T}\right)$$

Eq, (52)

Where the amplitude terms c, L, T are the speed of light, the quantization length and the characteristic time respectively. A plot of the input pulse is depicted in Fig.3. Optimizing this choice of input pulse for storage, one finds $\cos\theta(t)$ of the form:

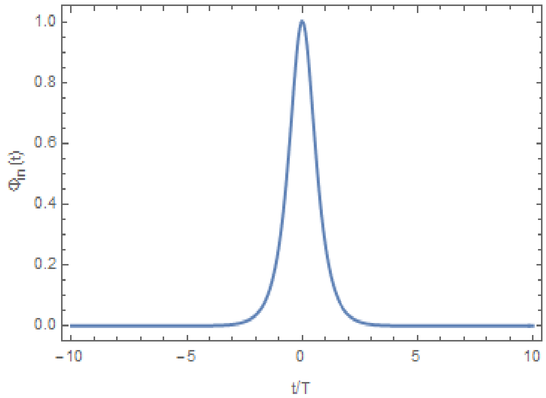

Figure 3. The input normalized hyperbolic secant pulse

$$\cos\theta(t)=\sqrt{\frac{2}{\gamma T}}\frac{\sec h\left(\frac{2t}{T}\right)}{\sqrt{1+\tanh\left(\frac{2t}{T}\right)}}.$$

Eq,(53)

where t_0 is such that the input amplitude is zero for all $t < t_0$ ($t_0 \rightarrow -\infty$). A plot of $\cos\theta(t)$ shows that $\cos\theta(t) \rightarrow 0$ as $t \rightarrow \infty$ which leads to the optimization of the input pulse (see fig.5). This corresponds to varying the Rabi frequency, eq. (50)

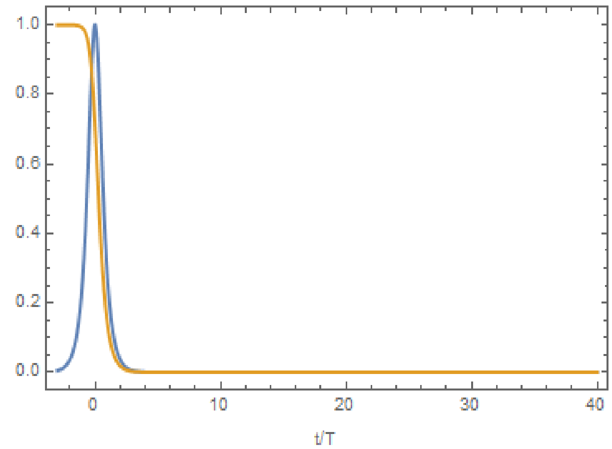

Figure 4: A plot of ($\cos\theta(t)$ (orange) showing the time optimization of the input field $\Phi_{in}(t)$ (blue). $\gamma T = 4$.

Leading to:

$$\Omega(t)=g\sqrt{N}\frac{\sec h\left(\frac{2t}{T}\right)}{\sqrt{\frac{\gamma T}{2}\left[1+\tanh\left(\frac{2t}{T}\right)\right]-\sec h^2\left(\frac{2t}{T}\right)}}.$$

Eq, (54)

This choice of varying $\Omega(t)$ leads to the adiabatic transfer into the cavity dark state. For t_0 such that the input amplitude is zero for all $t < t_0$ and $t_0 \rightarrow -\infty$, we obtain the dark state population to be of the form:

$$|D(t)|^2=\frac{1+\tanh\left(\frac{2t}{T}\right)}{2}. \qquad \text{Eq, (55)}$$

Thetransferofsingle-photonstatetothecavity-darkstateisdescribedbythekink soliton. Fig.6 shows the dark state population approaching unity as $t \rightarrow \infty$.

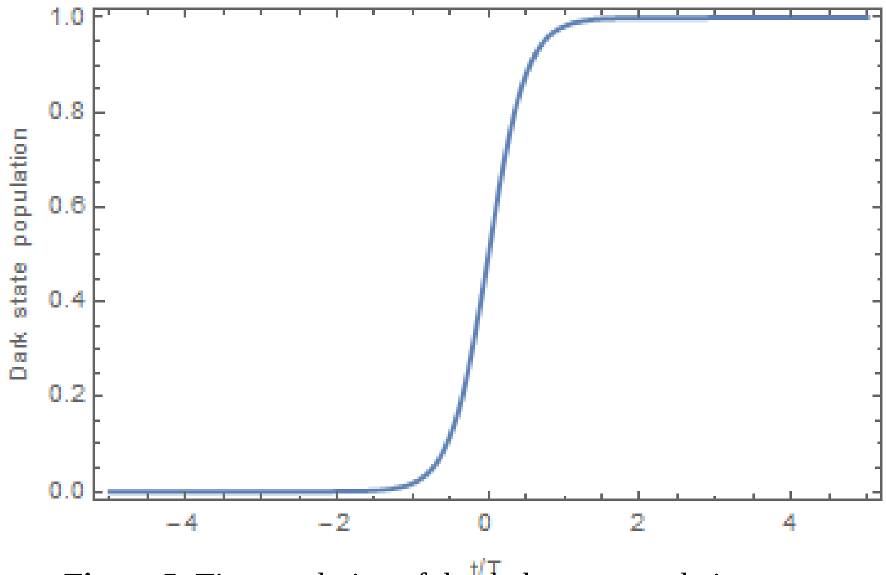

Figure 5: Time evolution of the dark-state population

In the fig. 5, it is observed that the curve of the dark state population tends close to unity implying that we obtain high probability of transference into dark state. The essential point of this technique is not for the purpose of storage of the energy or momentum carried by the photon but, rather the storage of the quantum state.

Output: Retrieval of the input pulse

Once the adiabatic storage into the cavity-dark state, it is relevant to be able to retrieve the stored wave packet. In order to release the stored photon into the free- field photons at some later time *t*, one can simply reverse the adiabatic rotation of the mixing angle.

For the retrieval process, consider a time t large enough such that $\Phi_{in}(0,t)=0$ for all $t>t_1$ and for $\cos(t_1)=0$, we find from eq. (45) the output as:

$$\Phi_{out}(0,t)=-\sqrt{\gamma \tfrac{L}{c}}D(t_1)\cos\theta\exp\left(-\frac{\gamma}{2}\int_{t_1}^{t}\cos^2\theta(\tau)\,d\tau\right)$$

Eq. (56)

This consist of adiabatically reversing the effect of $\Omega(t)$, this simplifies the above retrieved output field according to:

$$\Phi_{out}(t)=-\frac{\Phi_{in}(t)}{\int_{t_0}^{t}\Phi_{in}^2(t')\,dt'}\int_{t_0}^{t_1}\Phi_{in}^2(\tau)\,d\tau$$

Eq. (57)

The output for the choice of eq. (3.52) is given by:

$$\Phi_{out}=-\sqrt{\frac{L}{cT}}\left[1+\tanh(2t_1/T)\right]\frac{\sec h(2t/T)}{1+\tanh(2t/T)}$$

. Eq. (58)

From eq. (57) and eq. (58), one can verify that the general expression for the output field in terms of the inject input field is expressed as:

$$\Phi_{out}(t)=-\Phi(t)\left|\frac{D(t_1)}{D(t)}\right|^2.$$

Eq. (59)

Therefore, for a particular time of retrieval of $\Phi_{in}(t)$, one requires $t_1 \rightarrow t$. A plot for the output is shown in fig. 3.

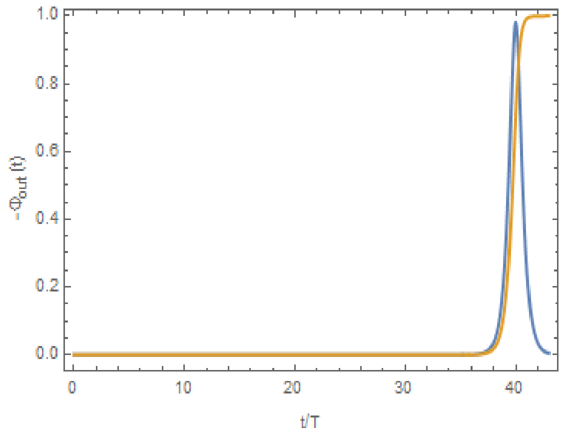

Figure 6: A plot of Φ_{out} (blue): Retrieval of stored pulse by time reversal of $\cos\theta(t)$ at $t = 40T$ (orange).

CONCLUSION

Having in mind the method presented in this paper, it was observed that the probe pulse (that is, of single-photon quantum states) can be stored in long live cavity dark state. The transfer of the single photon state to the collective cavity dark state can be accomplished with nearly a hundred percent efficiently by the method of intracavity EIT and assumption of quantum impedance matching condition whereby the classical driving field is adiabatically switched off while the probe laser is inside the dense ensemble of atoms and cos θ is optimized for any input pulse respectively. The retrieval was made possible by a reverse adiabatic process of switching on the control laser. It was concluded that, this method of using an optically-dense many-atom medium is far superior and efficient than the technique with single atoms processed in the strong coupling system.

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