MODELLING OF EXTENSIONAL FLOW BEHAVIOR OF POLYMERIC FLUID USING CAPILLARY THINNING EXPERIMENT

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ABSTRACT

The study presents the stretching behavior exhibits by the viscoelastic fluid with the scope of investigating how the stretching flow method can be predicted from simple mathematical models of polymeric fluids. However, the stretching flow technique is used to determine the extensional properties of polymer liquids. Our results show that the stretching behavior of the polymeric fluid causes a decline in radius. Moreover, we observed that the exponential decay curve plays a significant contribution in determining the relaxation times. This was deduced from the governing equations of the radially squeezed cylindrical fluid column with the use of distinct non-dimensional parameters obtained from the force balance and three relaxation times were achieved. The curve appears to be Newtonian at the initial stage and then changed to elastic and cut-off in the final stage. The motive for this study is to provide a significant understanding of fluid flow phenomenon of polymeric liquid by stretching techniques and computations. This flow phenomenon was investigated with the view of understanding the rheological viewpoint governing the stretching behavior in elastic fluids and the techniques employed in determining these flow responses to deformation.

Keywords: Capillary thinning; governing equation; extensional flow; viscoelastic flow.

INTRODUCTION

Polymeric fluids are characterized as non-Newtonian fluids which simply mean the fluids that possess dissimilar behavior in their flow properties from Newtonian fluids (Szabo et-al,. 2012). Examples of these fluids are toothpaste, ketchup, corn starch mixtures and others. Some of these substances are viscoelastic, that is, they possess both viscosity (a property of fluid) and elasticity (a property of solid). The rate at which a strain is applied on viscoelastic materials describes the behavior of the material's response to deformation (Clasen *et al.*, 2004). If the strain is applied slowly, the behavior at the after-effect will be fluid-like. In contrast, if the strain is applied quickly then the material shows solid-like behavior. Also non-Newtonian fluids are described with mechanical properties of polymer which has to do with extensional properties.

In this paper, we investigate and analyze the extensional flow using capillary rheometer with the aim of determining the extensional properties of polymeric fluids, exploring the rheological frame of reference governing the extensional behavior in polymeric fluids. Furthermore we analyze the techniques in determining the flow responses to deformation using the mechanical properties of polymeric fluids from the simple mathematical models, where a cylindrical column of elastic fluid undergoes extensional distortion by a surface tension (Entov et-al,.1997) with the view of analyzing the consequence of relaxation time in stretching position and scale between the viscous stage up to elastic stage period of the deformation. Also the break-up time in the elastic fluid that has been influenced by elastic stress at the elastic stage.

Related Literature

The stretching of such liquids produces a very long filaments or threads of fluids; this happens as a result of strong extensional resistance exhibit by their flow properties.
Extensional flow has a very significant value in polymer processing industries, giving a greater understanding of the integral extensional behavior of polymer mixtures especially in polymer processing plant. The flow deals with the stretching and deformation of polymer melts and solutions with no shear (Morrison, 2001). These liquids are known as non-Newtonian, simply means that they possess dissimilar behavior in flow properties from Newtonian liquids (Szabo et-al., 2012). Often polymer materials behaves differently depending on the type of flow is applied, quite commonly their shear thinning and extensional hardening.

The stretching of those materials produces very long filaments of fluid. This happens as a result of strong extensional resistance exhibits by their flow characteristics. These flows are rather a complex fluid flows, in the sense that the flow properties are very difficult to determine due to many unresolved defects exhibited by the flow fields. These occur due to some physical structures such as temperature differences, molecular structure and concentrations in those solutions and melts. External forces such as gravity, boundary conditions and surface tension also influences the flux reliability (Young et-al., 2011).

In spite of these complexities, many investigations were carried out to determine the extensional flow properties through the means of devices such as the filament stretching rheometers and many others. According to (Morrison, 2001) the apparatus used to perform the laboratory testing and experiments in order to determine the rheological phenomena are the Sridhar's Apparatus, the Load cell and the Meissner rheometer. The Sridhar's Apparatus is use for determining the extensional resistance to flow of polymer mixtures by means of stretching. This device is made up of two circular plates where a little amount of polymeric liquid specimen is situated between the plates and later the plates were moved apart in regular manner with the aim of determining the fluid thread distortion rate together with the load cell attached at the end of the motionless plate to compute the force in the fluid thread. Other devices attached to this experiment are Charged Coupled Device (CCD) camera, computer motion controller, servo motor, to monitor the proceedings (Szabo et-al,. 2012). This experiment was carried out in order to provide adequate and acceptable solutions to those complexities encountered in the process of deformation in the fluid.

GOVERNING EQUATIONS

A polymeric liquid of cylindrical form is squeezed radially with a powerful surface tension effect, where extra stress is applied in the z direction in order to stretch liquid sample in that the extension rate is no longer predetermined, but must be found as part of the solution. This process is shown in Figure 1.

For the stretching in the axial direction with extension rate $E(t)$, the decline in radius $a(t)$ at the interval $0 \le r \le a(t)$ give

$$\frac{da}{dt} = \frac{-E}{2} a. \qquad (1)$$

The solution of Eq. (1) yields,

$$a(t) = a_o \exp\left(\frac{-E}{2} t\right).$$

The uncoupled finitely extensible nonlinear elastic (FENE) dumbbell model needed for the stretching flow with the radial deformation $A_{rr}(t)$ and the axial deformation $A_{zz}(t)$ are set as

$$\frac{dA_{rr}}{dt} = -EA_{rr} - \frac{f}{\tau}(A_{rr} - 1) \qquad (2)$$

$$\frac{dA_{zz}}{dt} = 2EA_{zz} - \frac{f}{\tau}(A_{zz} - 1) \qquad (3)$$

Where τ is called the relaxation time and f is the FENE factors define by

$$f = \frac{L^2}{L^2 + 3 - A_{zz} - 2A_{rr}} \qquad (4)$$

L^2 is the finite extension. In Figure 1, at free surface, the exerted capillary force is identical with the stress in radial τ_{rr}, so that

$$\tau_{rr} = -p - \mu E + GfA_{rr} = \frac{-\chi}{a}.$$

The right-hand side is the stress due to surface tension. The axial stress is assumed to be zero as

$$\tau_{zz} = -p + 2\mu E + GfA_{zz} = 0.$$

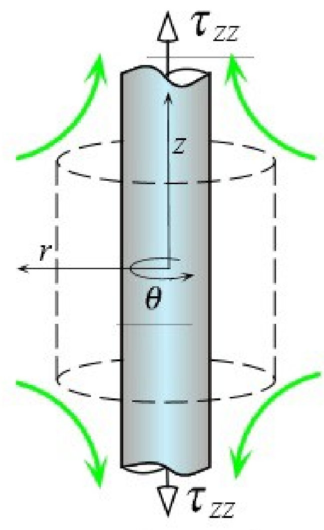

Figure 1: An illustration of a stretched polymeric fluid column in cylindrical form.

By removing p (pressure term) and subtracting the radial from the axial terms yields,

$$3\mu E + Gf(A_{zz} - A_{rr}) = \frac{\chi}{a} \qquad (5)$$

The viscous stress $3\mu E$ and the elastic stress $f(A_{zz} - A_{rr})$ give balance to the surface tension on the right-hand side. We need to find the strain rate $E(t)$ as part of the solution from the force balance in Eq. (5) so that

$$E = \frac{\chi}{3\mu a} - \frac{G}{3\mu} f(A_{zz} - A_{rr}).$$

For easier parameterization we multiply both sides by τ which gives

$$E\tau = \frac{\chi\tau}{3\mu a} - \frac{G\tau}{3\mu} f(A_{zz} - A_{rr}).$$

Defining two non-dimensional parameters $B = \frac{\chi\tau}{\mu a_o}$ the ratio of surface tension, $C = \frac{G\tau}{\mu}$ the ratio of polymer viscosity and solvent viscosity and choosing $D = E\tau$ therefore

$$D = \frac{Ba_0^{-1} - Cf\left(A_{zz} - A_{rr}\right)}{3} \qquad (6)$$

The Eq. (6) will be used in determining the numerical solution of the deformation and gradual developing of the fluid column. To find the decline in radius $a(t)$ from Eq. (5) in the absence of stress in viscous fluid term and the radial deformation, the equation reduces to

$$GA_{zz} = \frac{\chi}{a}.$$

This represents the balance between the axial stress and the surface tension. So that

$$a(t) = a_0\left(\frac{Ga_0}{\chi}\right)^{\frac{1}{3}} \exp\left(\frac{-t}{3\tau}\right). \qquad (7)$$

For the surface tension being substantial, we see that the form of a solution for the axial deformation decreases exponentially in time. At the early viscous stage which is the initial stage, the elastic stage is considered to be negligible, so therefore the analytical solution of the axial deformation, the radial deformation and the decline in radius are examine. The Eq. (5) reduces to the form

$$3\mu E = \frac{\chi}{a}.$$

We need to find the decline in radius of the fluid column by substituting for the extension rate in Eq. (1) and integrate the expression to yield

$$a(t) = a_0 - \frac{\chi}{6\mu}t. \qquad (8)$$

At the elastic stage, the stresses in the fluid increases towards the viscous stage ending, making the elastic stress start to be significant. Therefore the axial deformation is considered to be large as a result of the surface tension effect acting in order to stretch the fluid column. From Eq. (4) at equilibrium, the FENE factors $f \approx 1$. This transform the uncouple dumbbell equation to be in the form of the linear dumbbell model so that the axial deformation in Eq. (3) becomes

$$\frac{dA_{zz}}{dt} = 2EA_{zz} - \frac{1}{\tau}\left(A_{zz} - 1\right).$$

Taking $A_{zz} \geq 1$ (large), the initial condition at $t = 0$, $a = a_0$ gives

$$A_{zz}(t) = \left(\frac{a_0}{a}\right)^4 \exp\left(\frac{-t}{\tau}\right). \qquad (9)$$

This shows the exponential decay with time for the axial deformation. The same procedure applies to the radial deformation in Eq. (2) with the same boundary conditions to give

$$A_{rr}(t) = \left(\frac{a}{a_0}\right)^2 \exp\left(\frac{-t}{\tau}\right). \qquad (10)$$

We can however, examine the fluid response to deformation by putting Eq. (7) into the Eq. (9) to see the behavior of the axial deformation. This shows that

$$A_{zz}(t) = \left(\frac{Ga_0}{\chi}\right)^{\frac{-4}{3}} \exp\left(\frac{t}{3\tau}\right).$$

This implies that the axial deformation increase exponentially with the decline in radius. Now looking for the strain-rate, this can be calculated by substituting the prediction of the decline in radius in Eq. (7) with the initial decline in radius in Eq. (1). Hence

$$E = \frac{2}{3\tau}.$$

At the fully extended flow regime, after the elastic stress has been accumulated, the flow tend to become a steady extensional flow, thereby, the material derivative of the axial deformation in Eq. (3) turn out to be insignificant. This become

$$2EA_{zz} = \frac{f}{\tau}\left(A_{zz} - 1\right).$$

For $A_{zz} \geq 1$, we need to set $E\tau > \frac{1}{2}$ and $L^2 \geq 1$, this gives $2E\tau A_{zz} = fA_{zz}$. Using these set of conditions Eq. (4) becomes

$$2E\tau = \frac{L^2}{L^2 - A_{zz}}.$$

Therefore the prediction of axial deformation in the steady extensional flow gives.

$$A_{zz}(t) = L^2\left(1 - \frac{1}{2E\tau}\right) \quad (11)$$

RESULTS

The result shows stretching behaviour of the polymeric fluid at a predefined manner, in order to measure the transitional behaviour on the axial deformation, the radial deformation of the stretched fluid sample taking into account the surface tension acting on the fluid. Figure 2 indicates the behavior of the decline in radius after the stretching effect. Choosing the non-dimensional parameters $B = 5$ and $C = 1$, with FENE factors $f > 1$ in D. The solid line shows the exponential decay with time for Eq. (1), the dashed line represents Eq. (8), which shows linear decrease with time. For a fixed value of the parameter B, the linear dashed line remain unchanged for lower and higher values of C. Unlike the solid line, which keep decreasing exponentially with time at faster rate as the value of C increase. Close to the unity at some time interval, the region exhibit linear decay. The fluid shows a kind of Newtonian behaviour, afterwards eventually changes to exponential decay with time. As the axial deformation is increasing exponentially, so also the decline in radius is decaying exponentially showing the stretching effect.

Assuming the finite extension limits L is large, that is $L >> 1$, choosing the FENE factors $f > 1$ in D, with $B = 5$, $C = 1$ together with different ranges of $L = 10$, 20 and 30 respectively, then figure 3 shows the exponential decay behavior with different relaxation times for the different values of finite extension limits L. We can see that the curves are behaving in the same manner at earlier times for different values of L for the exponential decay. At later times, lower values of L decay much faster. For higher values of L, the exponential decay curves increases with time but having slight differences in the time intervals between the regimes. This implies that the higher values of finite extension limits, the slower decrease in the radius of the filament with time.

The comparison between the FENE factors shown in Figure 4, for decline in radius in Eq. (1) indicates the exponential decay in different phases. The solid curve represent a case where $f > 1$ and the dashed curve is a case for $f \approx 1$. The behavior on the curves looks the same at the initial stage. The solid curve relaxes faster at the time scale over the dashed curve, which later relaxes at a higher time interval different from the solid curve. For the FENE factor greater than unity, choosing $L = 10$, the stretching behavior of the fluid column in solid line shows the faster decay in time at shorter time interval compared to the FENE factor equal to unity represented in the dashed line, which slowly decreases exponentially with time at a higher time interval. In the plot, at the initial part, both the two cases possesses similar behavior on the exponential decay curve with some regions on the curves behaving in a linear decay manner, which appears much greater in the case where the FENE factors $f \approx 1$.

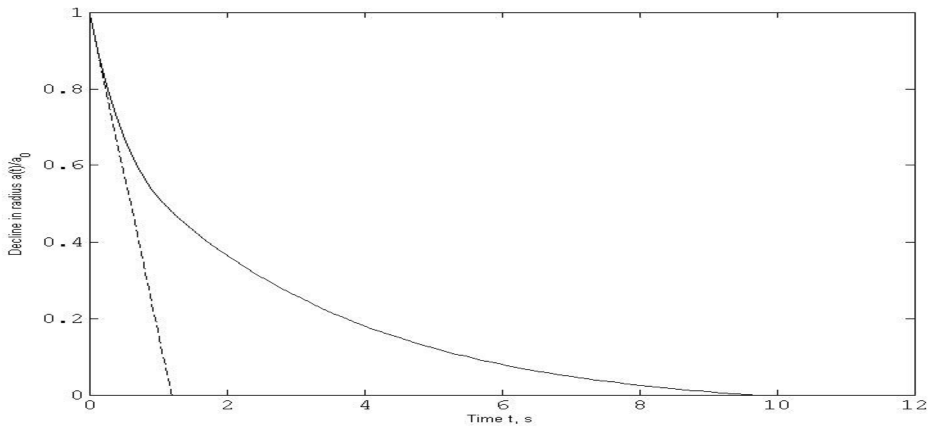

Figure 2: The decline in radius $a(t)/a_0$ with time, choosing the parameters $B = 5$ and $C = 1$.

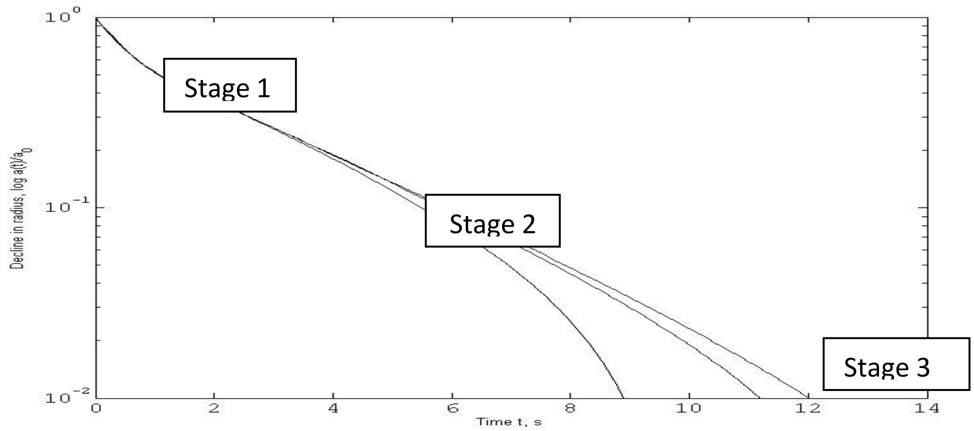

Figure 3: The decline in radius $a(t)/a_0$ with time with different values of finite extension slimits from left to right $L = 10$, 20 and 30.

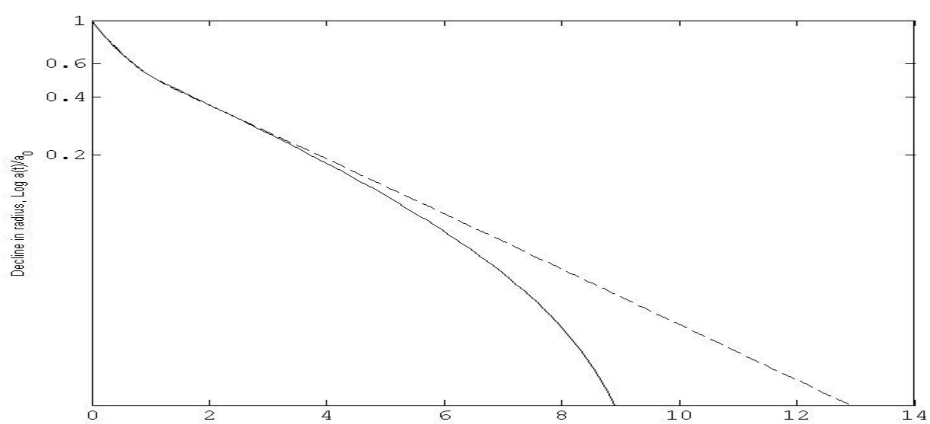

Figure 4: Plot of the decline in radius $a(t)/a_0$ with time. A comparison between the FENE factors $f > 1$ and $f \approx 1$ with $L = 10$, adopting the parameter $B = 5$ and $C = 1$.

DISCUSSION

Three stages of capillary thinning can be seen in Figure 3. The first stage is essentially the Newtonian where the axial stress is smaller compared to the viscous stress $GA_{zz} \leq \mu E$. The second stage is the elastic or elasto-capillarity balance. This is the other way round for the first stage where the viscous stress is smaller compared to the axial stress $\mu E \leq GA_{zz}$. But the FENE factors and the axial deformation is bigger than unity but less than the finite extension limit $1 \leq A_{zz} \leq L^2$. The final stage is the fully extended viscous. In this case, we essentially have the axial deformation is close to the finite extension limits. Systematically, the curve starts as a Newtonian then gradually change to elastic stage and then finally cut-off in the third stage. The third stage is sort of viscous again, so there is a viscous time scale with extensional viscosity.

CONCLUSION

The experimental approach in measuring the extensional properties of polymeric fluids were considered in understanding the filament stretching phenomenon and computations using set of well-defined fluid governing equations. This is done to estimate the analytical and numerical solutions of constant flow and flow like solutions for Newtonian and viscoelastic fluids. The exponential decay curve plays a significant contribution in determining the relaxation times. This was deduced from the governing equations of the radially squeezed cylindrical fluid column with the use of distinct non-dimensional parameters obtained from the force balance and three relaxation times were achieved. The curve appears to be Newtonian at the initial stage, and then changed to elastic and cut-off in the final stage.

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