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MODELLING AND STABILITY ANALYSIS OF GONORRHEA TRANSMISSION MODEL WITH TREATMENT AND CONDOM EFFICACY

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ABSTRACT

A deterministic gonorrhea model is designed and used to study the dynamics of transmission of gonorrhea disease. The positivity of the solution was determined to be positive with the positive initial data; the invariant region was also determined to be positively invariant. In the absent of disease, the disease free equilibrium point was determined and shown to be both locally and globally asymptotically stable when the associated basic reproduction R_0, is less than unity .Furthermore, the model has a unique endemic equilibrium point which is locally and globally asymptotically stable, whenever the basic reproduction is greater than unity. Sensitivity analyses and numerical simulation carried out shows that increase in the effective contact rate**,** which is the most sensitive parameter followed by progression rate from exposed class to the infected class and the relative risk of an infected persons been infective increase the burden of the disease in the community, while increase in condom efficacy, compliance of condom used and treatment of gonorrhea infected individual will reduce the disease burden and so it has a significant impact in controlling gonorrhea transmission in the community.

Keyword: Modelling; Gonorrhea; Condom

INTRODUCTION

Gonorrhea is a sexually transmitted disease (STD) which is caused by the bacteria Neisseria Gonorrhoeae. Many of those infected with gonorrhea have no symptoms, the most common symptoms in men includes: burning urination, discharge from the penis or testicular pain while in women it includes vaginal discharge, vaginal bleeding between period and pelvic pain. Gonorrhoea is spread through sexual contact with an infected person; this includes oral sex, vaginal sex or anal sex. According to world health organization (WHO), about 106.1 cases of gonorrhoea were reported (WHO 2012). Abstaining from sex, using condom during sex and being in a monogamous relationship are the best ways to prevent gonorrhoea. There are risk factors that may increase the risk of contracting gonorrhea which includes: younger age, multiple sex partners, previous gonorrhoea diagnosis and having other sexually transmitted diseases among others. Untreated gonorrhoea can lead to significant complications, such as infertility in both men and women, increased the risk of contracting other STD's and complications in babies. Adelani O. A. el. *al.,* (2016), studied a deterministic model of gonorrhea disease, they analysed the efficacy

of condom use, the local and global stability of the disease-free equilibrium was determined to be stable when the basic reproduction number is less than unity, and they also carried sensitivity analysis. Numerical simulation showed that increase in the rate of condom used reduces the number of infected individuals. Patrick N. and Onoja A. (2018), developed a model on the two-sex model for gonorrhoea dynamics incorporating treatment and condom used. They showed that the disease-free equilibrium is steady when the basic reproduction number is less than unity. Ibrahim I. A. and Sulaiman U. (2018), modified the model developed by Sacrifice N. K *el. al*., (2006) and studied the dynamics of gonorrrhea with natural immunity and treatment effects. However, their work does not include preventive measures such as condom used and its compliance

In our work, we developed and analysed a model for gonorrhea transmission, by incorporating treatment for infected individuals and some preventive measure such as condom efficacy and the compliance in the used of condom. We also include the relative risk of infection by an infected individual receiving treatment.

The paper is organised as follows: section 2, present the formulation of the model, basic properties of the model were verified in section 3, the equilibria of model (1) is obtained and analysed, the sensitivity analysis of the parameters are analysed in section 5, numerical simulation is presented in section 6, while section 7 and 8 presents the discussion and conclusion respectively.

MODEL FORMULATION

Assumption of the model

The following assumptions were made when developing the model

i. The members of the population interact freely.
ii. Only condom used was introduced as preventive measure.
iii. We assumed that there a natural death rate and an induced death rate as a result of the disease.
iv. Parameters and variables are considered non-negative.
v. We assumed that there is no permanent immunity to the disease.

Model description

The total population $N(t)$ is divided in to five (5) class namely:-The susceptible class $S(t)$ $E(t)$, is the exposed class $I(t)$, is the infected class $T(t)$, the treatment class and $R(t)$ the recovered class .The total population which denoted by, $N(t)$ is given as $N(t)=S(t)+E(t)+I(t)+T(t)+R(t)$.The susceptible class $S(t)$, become exposed to gonorrhoea when they come in contact with an infected individual at the rate β, with recruitment rate of π, while an exposed individual moves to the infected class when infected at the rate τ or return to the susceptible class at the rate σ due to the preventive measure such as condone used when in contact and natural immunity, an infected persons in the infected moves to the treatment class at the rate κ for treatment and with death induced by the disease at the rate

d , and $\gamma$is the rate at which an individual recovered after receiving treatment. Since there is no immunity for the disease, the rate at which recovered individual moves to the susceptible class at rate θ or moves to the exposed class when in contact with the infected person at the rate ψ .The associated force of infection is given as $\lambda = \frac{\beta(1-\alpha\nu)(I+\varepsilon T)}{N}$ where $0<\varepsilon<1$ is the modified parameter, which account the relative risk of person receiving treatment being infectious.

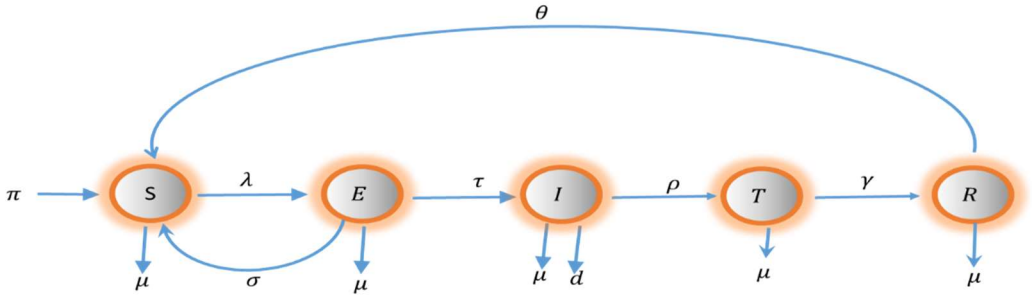

Figure 1. Compartmental diagram of the model

2.3 Model Equation

$$\frac{dS}{dt} = \pi + \sigma E + \theta R - \left(\mu + \frac{\beta(1-\alpha\nu)(I+\varepsilon T)}{N}\right)S \tag{1a}$$

$$\frac{dE}{dt} = \frac{\beta(1-\alpha\nu)(I+\varepsilon T)}{N}S - (\mu+\sigma+\tau)E \tag{1b}$$

$$\frac{dI}{dt} = \tau E - (\mu + d + \rho)I \tag{1c}$$

$$\frac{dT}{dt} = \rho I - (\gamma + \mu)T \tag{1d}$$

$$\frac{dR}{dt} = \gamma T - (\mu + \theta)R \tag{1e}$$

Table 1: Variables Description

Variable	Description
$S(t)$	Population of the susceptible class
$E(t)$	Population of the exposed individual
$I(t)$	Population of the infected individual
$T(t)$	Population of the infected receiving treatment
$R(t)$	Populations of those who recovered from Gonorrhea

Table 2: Parameters Description

Parameter	Description
π	Recruitment rate in the susceptible class
β	Effective contact rate
α	Condom efficacy
υ	Rate of compliance with the condone used
τ	Progression rate from the exposed class to the infected class
μ	Natural death rate
d	Disease induced death rate due to the disease
ρ	Progression rate from the infected class to the treatment class
γ	Probability of an individuals recovered from the disease after treatment
θ	Progression rate from the recovered class to the susceptible class
ψ	Progression rate from the recovered class to exposed class
σ	Progression rate from exposed to susceptible due to immunity or effect of condom efficacy
ε	Relative risk of infection when in contact with individual receiving treatment

Basic Properties of the Model

Positivity of the Solution

Theorem 1: Suppose that the initial data for the model be $S(0)>0,\ E(0)>0,\ I(0)>0$, $T(0)>0,$ and $R(0)>0$ then the solution $S(t),\ E(t),\ I(t),\ T(t)$ and $R(t)$ of the model with positive initial data will remain positive for all $t>0$.

Proof.

Let $t_1=\sup\{t>0: S(0)>0, E(0)>0, I(0)>0, T(0)>0, R(0)>0\}>0$ and from the first equation of the model 1, we have

$$\frac{dS}{dt}\geq \pi-(\mu+\lambda)S \qquad (2)$$

Integrating equation (2) using the integrating factor method we have

$$S(t_1)\geq S(0)\exp\left(-\mu t_1-\exp\left(\int_0^{t_1}\lambda(x)dx\right)\right)+\left\{\exp\left(-\mu t_1-\exp\left(\int_0^{t_1}\lambda(x)dx\right)\right)\right\}$$
$$\int_0^{t_1}\pi\exp\left(\mu y+\exp\left(\int_0^{t_1}\lambda(x)dx\right)\right)dy>0$$

Hence similarly it can be shown that $E(t_1)>0, I(t_1)>0\ T(t_1)>0$ and $R(t_1)>0$. Hence all the solutions of the model remain positive for all $t>0$.

The Invariant Region

Lemma 1: The closed set

$$\Omega = \left\{ (S,E,T,R) \in \Re_+^5;\ N \le \frac{\pi}{\mu} \right\}$$

is positively- invariant and attract all the positive solutions of the model

Proof

Adding equations in system (1), we have

$$N = \pi - \mu N - dI$$

$$\Rightarrow N \le \pi - \mu N \quad (3)$$

By the method of integrating factor, we integrate (3), which gives

$$N \le \frac{\pi}{\mu} + \left(N_0 - \frac{\pi}{\mu} \right) e^{-\mu t} \quad (4)$$

In particular if $N_0 \le \frac{\pi}{\mu}$, then $N \le \frac{\pi}{\mu}$. Hence Ω is positively invariant and an attractor so that no solution path leaves through any boundary of Ω.

Model Analyses

Disease Free Equilibrium Point (DFE) of the Model E_0

Setting $\frac{dS}{dt} = \frac{dE}{dt} = \frac{dI}{dt} = \frac{dT}{dt} = \frac{dR}{dt} = 0$, and since at DFE, $E = I = T = R = 0$, we have

$$E_0 = \left\{ S^*, E^*, I^*, T^*, R^* \right\} = \left\{ \frac{\pi}{\mu}, 0, 0, 0, 0 \right\}$$

Local Stability Analysis, (LAS) of the DFE, E_0

Basic Reproduction number R_0

Using the next generation matrix (P. Van den Driessche and James Watmough (2002)) where $R_0 = \rho(FV^{-1})$ where $F = \left[\frac{\partial F_i(x_0)}{\partial x_j} \right]$ and $V = \left[\frac{\partial V_i(x_0)}{\partial x_j} \right]$ or $i \ge 1$ for the number of compartments and $1 \le j \le m$ for the infected compartments only. $\rho(FV^{-1})$ denotes the spectral radius of the matrix. F and V are $m \times m$ matrices, where m is the number of infected classes, we have;

$$f = \begin{bmatrix} 0 & \frac{\beta(1-\alpha\upsilon)S^*}{N} & \frac{\beta\varepsilon(1-\alpha\upsilon)S^*}{N} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } v = \begin{bmatrix} (\mu+\sigma+\tau) & 0 & 0 \\ -\tau & (\mu+\rho+d) & 0 \\ 0 & -\gamma & (\mu+\gamma) \end{bmatrix} \text{ Hence}$$

$$R_0 = \frac{\beta\tau(1-\alpha\upsilon)[\mu+\gamma+\varepsilon\rho]}{(\mu+\sigma+\tau)(\mu+\rho+d)(\mu+\gamma)}$$

Lemma 2. The Disease Free Equilibrium point (DFE) E_0, of the Gonorrhea Model is LAS if $R_0 < 1$ and Unstable if $R_0 > 1$

Proof

The Jacobian matrix of the model is obtained at DFE and reduced using row elementary operation to upper triangular method as below

$$J_{E0} = \begin{bmatrix} -\mu & \sigma & -\tilde{\beta} & -\tilde{\beta}\varepsilon & \theta \\ 0 & -(\mu+\sigma+\tau) & \tilde{\beta} & \tilde{\beta}\varepsilon & 0 \\ 0 & 0 & -\dfrac{(\mu+\sigma+\tau)(\mu+\rho+d)-\tilde{\beta}\tau}{(\mu+\sigma+\tau)} & \dfrac{\tilde{\beta}\varepsilon\tau}{A} & 0 \\ 0 & 0 & 0 & \dfrac{(\mu+\sigma+\tau)(\mu+\rho+d)(\mu+\gamma)(R_0-1)}{(\mu+\sigma+\tau)(\mu+\rho+d)-\tilde{\beta}\tau} & 0 \\ 0 & 0 & 0 & 0 & -(\mu+\theta) \end{bmatrix}$$

where $\tilde{\beta} = \beta(1-\alpha\nu)$

Hence the eigenvalues of $|J_{E_0} - \lambda I| = 0$ are all negative if $R_0 < 1$ and $(\mu+\sigma+\tau)(\mu+\rho+d) > \tilde{\beta}\tau$, hence the DFE is LAS.

Global Stability (GAS) of the DFE of the Gonorrhea model E_0

Theorem 2: The disease free equilibrium point E_0, of the model is globally asymptotically stable (GAS) when $R_0 < 1$ and unstable if otherwise.

Proof

Consider the Lyponuv function

$$V = R_0 E + \frac{\beta(1-\alpha\nu)[\mu+\gamma+\varepsilon\rho]}{(\mu+\rho+d)(\mu+\gamma)} I + \frac{\beta(1-\alpha\nu)\varepsilon}{(\mu+\gamma)} T$$

Whose derivative is given as

$$\dot{V} = R_0 E' + \frac{\beta(1-\alpha\nu)[\mu+\gamma+\varepsilon\rho]}{(\mu+\rho+d)(\mu+\gamma)} \dot{I} + \frac{\beta(1-\alpha\nu)\varepsilon}{(\mu+\gamma)} \dot{T}$$

$$\dot{V} = R_0 \left(\lambda S - (\mu+\sigma+\tau)E\right) + \frac{\beta(1-\alpha\nu)[\mu+\gamma+\varepsilon\rho]}{(\mu+\rho+d)(\mu+\gamma)} \left(\tau E - (\mu+\rho+d)I\right) + \frac{\beta(1-\alpha\nu)\varepsilon}{(\mu+\gamma)} \left(\rho I - (\mu+\gamma)T\right)$$

Simplifying the above equation gives

$$\dot{V} = R_0\left(\frac{\beta(1-\alpha\nu)(I+\varepsilon T)S}{N}\right) - \beta(1-\alpha\nu)I - \beta(1-\alpha\nu)\varepsilon T$$

at $S^* \le N$ we have

$$\dot{V} \le \beta(1-\alpha\nu)(I+\varepsilon T)(R_0-1)$$

Hence $\dot{V} \le 0$ if $R_0 \le 1$ and $\dot{V} = 0$ if $I = T = 0$. Since all the parameters are non-negative it follows that $\dot{V} = 0$ if and only if $I = T = 0$. Hence V is a Lyapunov function on Ω. Furthermore, the largest compact invariant set in $\left\{\left(S^*(t), E^*(t), I^*(t), T^*(t), R^*(t)\right) \in \Omega : \dot{V} = 0\right\}$ is the singleton set $\{E_0\}$. Therefore by LaSalle's Invariant Principles every solution to the model with initial condition in Ω, approaches E_0 as $t \to \infty$ whenever $R_0 < 1$ so that E_0 is GAS in Ω if $R_0 < 1$.

Existence of the Endemic Equilibrium Point EEP E_1

Setting $\frac{dS}{dt} = \frac{dE}{dt} = \frac{dI}{dt} = \frac{dT}{dt} = \frac{dR}{dt} = 0$ we have the endemic equilibrium point as follows

$$S^{**} = \frac{\pi(\mu+\sigma+\tau)(\mu+\rho+d)(\mu+\gamma)(\mu+\theta)}{K}$$

$$E^{**} = \frac{\pi\lambda^{**}(\mu+\theta)(\mu+d+\rho)(\mu+\gamma)}{K}$$

$$T^{**} = \frac{\pi\lambda^{**}\tau\rho(\mu+\theta)}{K} \qquad (6)$$

$$I^{**} = \frac{\pi\lambda^{**}\tau(\mu+\theta)(\mu+\gamma)}{K}$$

$$R^{**} = \frac{\pi\lambda^{**}\tau\rho\gamma}{K}$$

where

$$K = (\mu+\sigma+\tau)(\mu+\rho+d)(\mu+\gamma)(\mu+\theta)(\mu+\lambda^{**}) - \lambda^{**}\{\sigma(\mu+\rho+d)(\mu+\gamma)(\mu+\theta)+\theta\gamma\rho\tau\}$$

Given the force of infection in terms of the EEP, we have

$$\lambda^{**} = \frac{\beta(1-\alpha\nu)\left(I^{**} + \varepsilon T^{**}\right)}{N^{**}} \tag{7}$$

Substituting (6) in (7) we have

$$a\lambda^{**} + b = 0$$

where $a = (\mu+\rho+d)(\mu+\gamma)(\mu+\theta+\psi)+\tau(\mu+\gamma)(\mu+\theta+\psi)+\rho\tau(\mu+\theta+\psi)+\rho\gamma\tau$ and $b = \left((\mu+\sigma+\tau)(\mu+\rho+d)(\mu+\gamma)(\mu+\theta+\psi)-\gamma\psi\rho\tau\right)(1-R_0)$, it is clear that $a > 0$ and $b < 0$,when $R_0 > 1$, hence the system has a unique positive solution given by $\lambda^{**} = -\frac{b}{a}$ when $R_0 > 1$.

Hence the following

Theorem 3: The Gonorrhea model (1), has a unique endemic (of the form E_1) whenever $R_0 > 1$ and no endemic equilibrium point when otherwise.

Theorem 4: The Endemic Equilibrium Point (EEP) E_1, of the model is Locally Asymptotically Stable (LAS) if $R_0 > 1$ and unstable if otherwise.

Proof

The proof is based on transforming the problem of analyzing the stability of an equilibrium points to that of analyzing the stability of a fixed point. From equation (7) gives the fixed problem of the form

$$f\left(\lambda^{**}\right) = \frac{\beta(1-\alpha\nu)D\tau\left(C+\varepsilon\rho\right)\lambda^{**}}{ABCD+\left(BCD+CD\tau+D\rho\tau+\rho\gamma\tau\right)\lambda^{**}}$$

whose derivative is as follows

$$f'\left(\lambda^{**}\right) = \frac{\beta D\tau(1-\alpha\nu)(C+\varepsilon\rho)\left(ABCD+\left(BCD+CD\tau+D\rho\tau+\rho\gamma\tau\right)\lambda^{**}\right)}{\left(ABCD+\left(BCD+CD\tau+D\rho\tau+\rho\gamma\tau\right)\lambda^{**}\right)^2} - \frac{\beta D\tau\lambda^{**}(1-\alpha\nu)(C+\varepsilon\rho)\left(BCD+CD\tau+D\rho\tau+\rho\gamma\tau\right)}{\left(ABCD+\left(BCD+CD\tau+D\rho\tau+\rho\gamma\tau\right)\lambda^{**}\right)^2}$$

where $A = (\mu+\sigma+\tau)$ $B = (\mu+\rho+d)$ $C = (\mu+\gamma)$ and $D = (\mu+\theta)$

$$= \frac{\beta ABCD^2\tau(1-\alpha\nu)(C+\varepsilon\rho)}{\left(ABCD+\left(BCD+CD\tau+D\rho\tau+\rho\gamma\tau\right)\lambda^{**}\right)^2}$$

Evaluating $f'\left(\lambda^{**}\right)$ at $\lambda^{**} = -\frac{b}{a}$

where $a=\left(BCD+CD\tau+D\rho\tau+\rho\gamma\tau\right)$ and $b=ABCD\left(1-R_0\right)$, we have

$$f'\left(\lambda^{**}\right)=\frac{\beta\tau\left(1-\alpha\nu\right)\left(C+\varepsilon\rho\right)ABCD^2}{\left(ABCD+ABCD\left(1-R_0\right)\right)^2}$$

$$=\frac{ABCD^2\beta\tau\left(1-\alpha\nu\right)\left(C+\varepsilon\rho\right)}{\left(ABCDR_0\right)^2}$$

$$=\frac{1}{R_0}$$

Hence, we clearly see that

$$\left|f'\left(\lambda^{**}\right)_{\lambda^{**}=-\frac{b}{a}}\right|<1$$

Whenever $R_0>1$. Thus, the unique endemic equilibrium point is locally asymptotically stable when $R_0>1$.

Global Stability (GAS) of the EEP, E_1 of the Gonorrhea Model

Theorem 5. The endemic equilibrium point E_1, of the Gonorrhea model (1) is globally asymptotically stable (GAS) if $R_0>1$ and $S\le S^{**}$.

Proof

Consider the function

$$V=\left(S-S^{**}-S\ln\frac{S^{**}}{S}\right)+\left(E-E^{**}-E^{**}\ln\frac{E^{**}}{E}\right)+\left(\frac{\left(\mu+\sigma+\tau\right)-\sigma}{\tau}+\frac{\sigma S^{**}}{\tau S}\right)\left(I-I^{**}-I^{**}\ln\frac{I^{**}}{I}\right)$$
$$+\frac{\tilde{\beta}\varepsilon S^{**}}{\left(\mu+\gamma\right)}\left(T-T^{**}-T^{**}\ln\frac{T^{**}}{T}\right)$$

whose derivative is

$$V'=\left(1-\frac{S^{**}}{S}\right)S'+\left(1-\frac{E^{**}}{E}\right)E'+\left(\frac{\left(\mu+\sigma+\tau\right)-\sigma}{\tau}+\frac{\sigma S^{**}}{\tau S}\right)\left(1-\frac{I^{**}}{I}\right)I'+\frac{\tilde{\beta}\varepsilon S^{**}}{\left(\mu+\gamma\right)}\left(1-\frac{T^{**}}{T}\right)T' \quad (15)$$

Substituting the values S',E',I',T' in (15), we have

$$V' = \left(1 - \frac{S^{**}}{S}\right)\left(\pi + \sigma E + \theta R - \left(\mu + \tilde{\beta} I + \tilde{\beta}\varepsilon T\right)S\right) + \left(1 - \frac{E^{**}}{E}\right)\left(\left(\tilde{\beta} I + \tilde{\beta}\varepsilon T\right)S - \left(\mu + \sigma + \tau\right)E\right)$$
$$+ \left(\frac{\left(\mu + \sigma + \tau\right) - \sigma}{\tau} + \frac{\sigma S^{**}}{\tau S}\right)\left(1 - \frac{I^{**}}{I}\right)\left(\tau E - \left(\mu + d + \rho\right)I\right) + \frac{\tilde{\beta}\varepsilon S^{**}}{\left(\mu + \gamma\right)}\left(1 - \frac{T^{**}}{T}\right)\left(\rho I - \left(\gamma + \mu\right)T\right)$$

Further Simplification and Suppose $S \le S^{**}$ then we have

$$V' \le \frac{\theta R S^{**}}{S}\left(\frac{S}{S^{**}} - 1\right) + \frac{\theta R^{**} S^{**}}{S}\left(1 - \frac{S}{S^{**}}\right) + \sigma E^{**}\left(1 - \frac{S}{S^{**}}\right) + \frac{\sigma E S^{**} I^{**}}{IS}\left(\frac{S}{S^{**}} - 1\right) + \mu S^{**}\left(2 - \frac{S}{S^{**}} - \frac{S^{**}}{S}\right)$$
$$+ \tilde{\beta} S^{**} I^{**}\left(3 - \frac{S^{**}}{S} - \frac{SIE^{**}}{EI^{**}S^{**}} - \frac{EI^{**}}{IE^{**}}\right) + \tilde{\beta}\varepsilon S^{**} T^{**}\left(4 - \frac{S^{**}}{S} - \frac{TSE^{**}}{ES^{**}T^{**}} - \frac{I^{**}E}{E^{**}I} - \frac{IT^{**}}{TI^{**}}\right)$$

Since the arithmetic mean exceeds the geometric mean the following inequality hold: $1 - \frac{S}{S^{**}} \le 0$, $2 - \frac{S}{S^{**}} - \frac{S^{**}}{S} \le 0$, $3 - \frac{S^{**}}{S} - \frac{SIE^{**}}{EI^{**}S^{**}} - \frac{EI^{**}}{IE^{**}} \le$ and $4 - \frac{S^{**}}{S} - \frac{TSE^{**}}{ES^{**}T^{**}} - \frac{I^{**}E}{E^{**}I} - \frac{IT^{**}}{TI^{**}} \le 0$

Furthermore, since all the model parameters are non-negative, thus $V' \le 0$, then the largest compact invariant set in Ω_1 such that $V' \le 0$ is the singleton set $\{E_1\}$ then by LaSalle Invariant Principle then it implies E_1 is globally asymptotically stable (GAS) in the interior of Ω_1.

SENSITIVITY ANALYSIS

Sensitivity analysis is very important in determining how sensitive a model is to change in the value of the parameter of the model. It is used to discover parameter that have light on R_0 and should be targeted by intervention strategies. The normalized forward sensitivity index of the gonorrhea reproduction number R_0 to a parameter is the relative change in the variable R_0 to the relative change in a given parameter. A direct proportional normalized sensitivity index indicates that an increase/decrease in the parameter value brings about an increase/decrease respectively in the value of R_0. whereas an inversely proportional normalized sensitivity index indicates that an increase in parameter value bring about a decrease in the value of R_0.

Definition: let $R_0 : U \to Z$ and $R_0 \in C'(U)$ where $(U, Z) \subseteq \Re^+$ for all parameter $q \in U$, the normalized forward sensitivity index is defined as

$$\Upsilon_q^{R_0} = \frac{\partial R_0}{\partial q} \cdot \frac{q}{R_0}$$

Using the formula for the reproduction number obtained in (3.6.1), we have

$$\Upsilon_\beta^{R_0} = \frac{\partial R_0}{\partial \beta} \cdot \frac{\beta}{R_0} = +1$$

Applying the formula for rest of the parameter in R_0 and the results are tabulated below.

Table 3: Sensitivity Indices of R_0 with respect to each parameter

Parameter	Sensitivity Index
β	+1
α	-0.4285714
υ	-0.4285714
τ	0.5073314
ρ	-1.1854693
γ	-0.2168541
ε	0.3144385
σ	-0.5367412

Numerical simulations

The numerical simulation is carried out using MATLAB R2012b and with parameter values obtained in Table 3 above. The values of β, α and ν were varied to different values so as to show the effect of the parameters on the model.

Table 4: Parameter Values

Parameter	Value	Source
π	800	Estimated
β	0.8	Estimated
α	0.5	Estimated
ν	0.6	Estimated
τ	0.6	Ibrahim I and sulaiman U (2018)
μ	1/56	Estimated
d	0.02	Estimated
ρ	0.5	Ibrahim I and sulaiman U (2018)
γ	0.5	Ibrahim I and sulaiman U (2018)
θ	0.7	Ibrahim I and sulaiman U (2018)
σ	0.6	Estimated
ε	0.3	Estimated

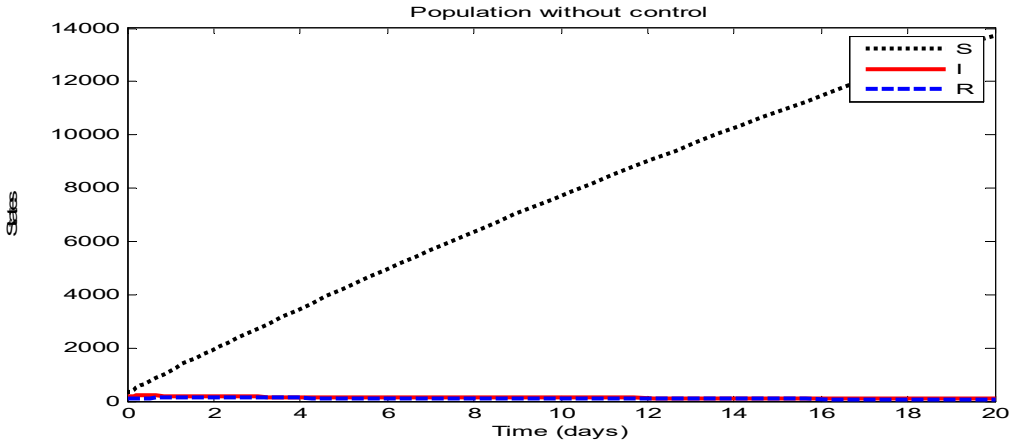

Figure 1: Simulation Results for Susceptible, Infected and Recovered Class when $\beta = 0.8$, $\alpha = \nu = 0$ with $R_0 = 0.9450434$

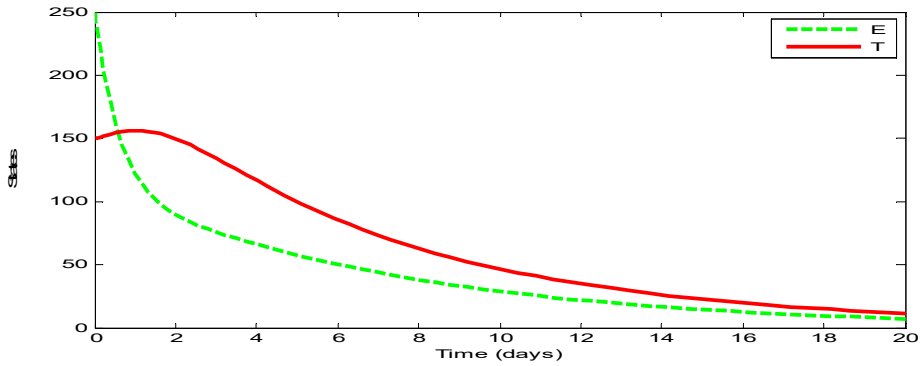

Figure 2: Simulation Results for Exposed and Treatment Class when $\beta = 0.5$, $\alpha = \nu = 0$ with $R_0 = 0.5906521$

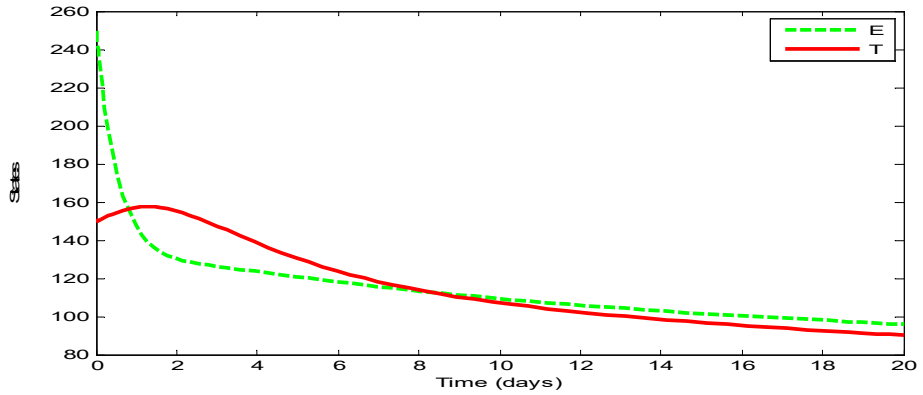

Figure 3: Simulation Results for Exposed and Treatment Class when $\beta = 0.8$, $\alpha = \nu = 0$ with $R_0 = 0.9450434$.

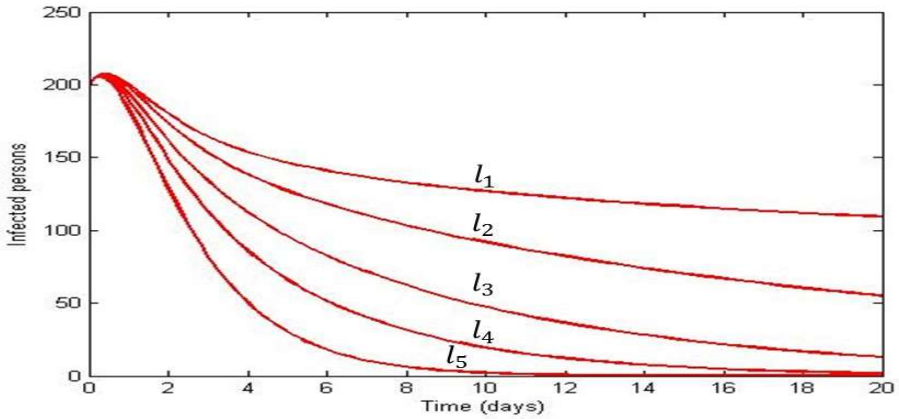

Figure 4: Simulation Results for Infected Class when, $\beta = 0.8$ $\alpha = [0, 0.25, 0.5, 0.75, 1]$ and $\nu = [0, 0.5, 0.7, 0.8, 1]$

DISCUSSION

The epidemiological implication of lemma 2, is that gonorrhea can be effectively controlled in the community (when $R_0 < 1$), if the initial size of the sub-population of the model are in the basin of attraction of the DFE, and GAS of the DFE implies that the requirement $R_0 < 1$ is necessary and sufficient for the effective control or elimination of the gonorrhea disease. The EEP is also determined and from lemma 3, it was shown to be LAS if $R_0 > 1$; the implication is that the disease will persist when $R_0 > 1$ and $S \le S^{**}$ for $t \ge 1$, for the EEP to be globally asymptotically stable. Figure 1, shows that an increase in effective contact rate increases the population of the susceptible which implies that the disease can be control (given $R_0 = 0.9450434 < 1$). Table 2, Figure 2 and Figure 3, we see that increase in effective contact rate increases the number of the exposed and treated population and vice versa (since at $\beta = 0.5$ we obtained $R_0 = 0.5906521 < 1$ also at $\beta = 0.8$, we have $R_0 = 0.9450434 < 1$), since reduction in R_0 signifies a reduction in the disease burden. In Figure 4, line l_1 is obtained when $\alpha = \nu = 0$, that is, when condom efficacy is zero and using the parameter values in Table 4, line l_2 is obtained when $\alpha = 0.25$ and $\nu = 0.5$, line l_3 is obtained when $\alpha = 0.5$ and $\nu = 0.7$, line l_4 is obtained when $\alpha = 0.75$ and $\nu = 0.8$, and line l_5 is obtained when $\alpha = \nu = 1$,which revealed that the population of infected individuals decrease when condom efficacy and compliance in condom used increases, and hence an increase in condom efficacy and compliance of condom used will reduce the disease burden. Table 2, shows that an increase in treatment of gonorrhea infected individual have a significant impact in controlling gonorrhea in the community.

CONCLUSION

Gonorrhea transmission model is developed and analysed. The model targets the main transmission route (i.e. sexually) and thus incorporate condom efficacy and its compliance. The two equilibria (DFE and EEP) were obtained and both are locally and globally asymptotically stable, furthermore the basic reproduction number was also obtained. Sensitivity analyses of the model parameters revealed that the effective contact rate β, is the most sensitive parameter followed by progression rate from exposed class to the infected class τ and ε, the relative risk of an infected persons been infective, while condom efficacy (α), compliance in condom use (ν), the progression rate from infected class to treatment class (ρ), the rate at which an infected persons recovered (γ) and progression rate from exposed to susceptible class (σ), have reciprocal impact (i.e. increase α, γ, α and ρ decrease the basic reproduction). The numerical simulation of the model with parameter values in Table4 reveals the same result with the sensitivity analysis.

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