Bima Journal of Science and Technology, Vol. 3(1) July, 2019. ISSN: 2536-6041

MODELING THE DYNAMIC OF KALARE CRIME IN GOMBE METROPOLIS

*SANDA AYUBA, ADAMU IBRAHIM, YAHAYA AJIYA, ADAMU ISHAKU, IBRAHIM GARBA AND ALIYU ABUBAKAR

Department of Mathematics, Gombe State University, Gombe, Nigeria
*Corresponding Author: sandaayuba@gmail.com

ABSTRACT

The activities of kalare youth group have put an end to the lives of several innocent people of Gombe, the state capital of Gombe, Nigeria more than any other kind of crime in the recent history of the state. In this paper, a mathematical model of kalare crime with two categories of susceptible class was developed and analyzed. We obtained the existence and uniqueness of solution of the model equations, the basic reproduction number R_0 best on the modification of the work of Umar (2013). The analysis shows the crime-free equilibrium is locally asymptotically stable whenever the threshold $R_0 < 1$ and unstable if $R_0 > 1$. The crime present equilibrium of the model exists and it is found to be unique under certain condition. The numerical simulation carried on three different scenarios as $R_0 = 0.27714 < 1$, $R_0 = 0.20123 < 1$ and $R_0 = 0.0694 < 1$ which shows that kalare crime can be drastically reduce.

Keywords: Crime, Kalare, Dynamic, Gombe, Stability, Population

INTRODUCTION

Kalare is a name of a gang star who is a hunter from a village called Miya in Bauchi. Kalare is well known, who usually attends occasion organize by his fellow hunters anywhere in the northern part of Nigeria. Sometimes back in 1994 the hunters of Bolari district in Gombe, organize an occasion led by the District Head of Bolari as Barde, and Yan Dawa which they invited kalare, due to his visit their name was changed from Barde, and Yan Dawa to Yan kalare. Yan kalare they are known to be hunters and they live in the bush unless during their annual festival until recently in 2003 general election when the politician invites them to serve as their guard, whenever they are embarking in campaign. As a result of that some jobless youth of Gombe joined them thereby involving themselves in one form of a political violence or the other, Umar (2013).

Short et al (2008) introduced the existence and stability of localized patterns of criminal activity for the two-components reaction-diffusion model of urban crime, such patterns, characterized by the concentration of criminal activity in localized spatial regions, are referred to as hot-spot patterns and they occur in a parameter regime far from the Turing point associated with the bifurcation of spatially uniform solutions. Nuno et al (2011) proposed and analyzed a mathematical model of a criminal-prone self-protected society and Sayantani and Arnab

(2017) modified the interaction function between criminals and guards in the work of Nuno et al (2011) model and investigated the impacts of this modification. The numerical simulation reported that the free-of-criminals steady state becomes asymptotically stable when the rate of crime is decreased (that is when the value of $k < 1$).

Compartmental models have been used to study the influence of sociological and economic factors on the evolution of criminality. Such studies will be useful to determine possible strategies for reducing and controlling crime. Criminal behavior and violence can also be modelled by using infectious disease model, Hochberg (1991). Agent-based model may also be suitable to investigate spread of criminal behavior (Hochberg (1991); Diekman et al (1990); Van et al (2002); and Adamu et al (2018)).

In this paper, we formulated a model to study the dynamic of this crime and by incorporating jail and rehabilitation.

MODEL DESCRIPTION

The population is divided into six compartments; the susceptible of educated S_E, the susceptible of uneducated S_U, the exposed class E, the crime class C, the jail class J, the recovery compartment R, and total population N is equal to the sum of the compartments. The susceptible of educated compartment is the class where people are capable of being expose to kalare crime due to illicit act by the kalare individual onto the susceptible class of educated people that is when a kalare gang attack one of their relative of the susceptible class of the educated compartment, the victim or the relative may go for revenge in the process of doing such they may be addicted to it with recruitment rate constant π_1 while the susceptible for uneducated compartment is the class where people are capable of being exposed to kalare crime due to contact rate with the kalare crime individuals, these include people who have closed relationship with the kalare crime individual and those that live in the same area with the kalare crime individuals with the recruitment rate constant π_2. The crime spread through contact between the crime individuals and the susceptible individual, the rate of change of population is proportional to the total number of the contacts, the number of contact between the susceptible of educated individual is proportional to the product of S_E and C, this implies there is a contact rate β_1 that define the rate at which the susceptible class of educated becomes exposed to kalare crime, while the number of contact between the susceptible of uneducated is proportional to the product of S_U and C, which implies there is positive contact rate β_2 that defined the rate at which susceptible of uneducated becomes exposed. The exposed compartment, are individual that are expose to kalare crime but do not encourage people to practice the kalare crime and they move into the crime compartment at rate α with the natural death rate of μ and death due to the crime at rate d_1 after the individual has become addicted with the kalare crime, and they also go to the jail compartment at rate θ_1 The crime class consist of those who practice kalare crime and capable of influencing other people to join the kalare crime either willingly or not. A proportion of this class may experience one of the four outcomes, they either die as a result of the crime with rate constant rate d_2 or death induce by natural course at rate μ or they go to the jail at rate θ_2, they also recover at rate τ and eventually move into the recovered compartment this compartment also has natural death rate of μ, While the jail compartment consist of those who practice kalare crime and they move into the recovery class with rate θ_3 and natural death rate μ or they move to the crime class with rate θ_4.

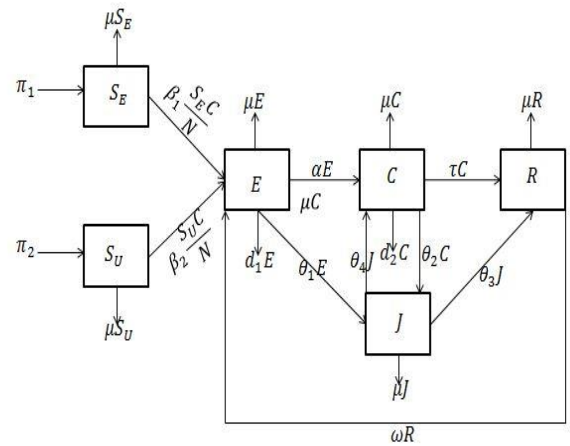

Figure 1: Compartmental diagram of the model

Model Equation

The Model Equation

Using the schematic diagram, we obtained the equation for the dynamic of kalare crime in Gombe as follows:

$$\frac{dS_E}{dt} = \pi_1 - \mu S_E - \beta_1 \frac{S_E C}{N} \quad (1)$$

$$\frac{dS_U}{dt} = \pi_2 - \mu S_U - \beta_2 \frac{S_U C}{N} \quad (2)$$

$$\frac{dE}{dt} = \beta_1 \frac{S_E C}{N} + \beta_2 \frac{S_U C}{N} + \omega R - \mu E - d_1 E - \alpha E - \theta_1 E \quad (3)$$

$$\frac{dC}{dt} = \alpha E + \theta_4 J - \mu C - \tau C - \theta_2 C - d_2 C \quad (4)$$

$$\frac{dJ}{dt} = \theta_1 E + \theta_2 C - \mu J - \theta_3 J - \theta_4 J \quad (5)$$

$$\frac{dR}{dt} = \tau C + \theta_3 J - \mu R - \omega R^* \quad (6)$$

Model Assumptions

Base on the model described above the following assumption are made:

1. In the beginning of the model, the total population N is the same as the sum of the number of susceptible individual, the exposed class, the crime class, the jail class and the recovery class.
2. The population is heterogeneous, that is the individual that make up the population can be categorized into different compartment according to their epidemiological state.
3. The population size in a compartment is differentiable with respect to time (t) and that the crime present equilibrium process is deterministic.
4. All susceptible individuals are equally likely to be crime present individuals when they come in contact.
5. Those in uneducated class are more vulnerable to be infected.
6. We assumed that there is an induced death rate as a result of the crime.
7. People in each compartment have equal natural death rate of µ.

Model Analysis

In this section, we obtained equilibrium state, the basic reproduction number, the stability of both crime-free and addiction equilibrium point and numerical solution were obtained.

Existence and Uniqueness of Solution

We formulate a theory on the existence and uniqueness of solution of model system of equations (1)-(6) following the approach of Adamu et al (2018) and we now establish the proof.

Theorem 1: Let D denote the region $0 \leq N \leq R$, then (1) have a unique solution.

We show that $\frac{\partial f_i}{\partial x_j}, i, j = 1, 2, \ldots, 6$ are continuous and bounded in D

Proof:

let $\frac{dS_E}{dt} = f_1, \frac{dS_U}{dt} = f_2, \frac{dE}{dt} = f_3, \frac{dC}{dt} = f_4, \frac{dJ}{dt} = f_5, \frac{dR}{dt} = f_6,$ respectively.

$$\frac{\partial f_1}{\partial S_E} = \left| -\mu - \frac{\beta_1 C}{N} \right| < \infty, \quad \frac{\partial f_1}{\partial S_U} = 0 \quad \frac{\partial f_1}{\partial E} = 0 \quad \frac{\partial f_1}{\partial S_E} = \left| \frac{\beta_1 S_E}{N} \right| < \infty, \quad \frac{\partial f_1}{\partial J} = 0 \text{ and } \frac{\partial f_1}{\partial R} = 0 < \infty \tag{7}$$

As clearly shown above, the partial derivative of the whole system exists, and they are finite and bounded. Similarly, by theorem 1, the model system (1)-(6) has a unique solution.

The Invariant Region

Invariant region is the region that makes the system to be more biologically sense.

$$\frac{dN}{dt} = \frac{dS_E}{dt} + \frac{dS_U}{dt} + \frac{dE}{dt} + \frac{dC}{dt} + \frac{dJ}{dt} + \frac{dR}{dt} \tag{8}$$

$$\frac{dN}{dt} \leq \pi_1 + \pi_2 - \mu N \tag{9}$$

$$\frac{dN}{dt} \leq \pi - \mu N \tag{10}$$

where $\pi = \pi_1 + \pi_2$. Integrating (10), we have

$$N \leq \frac{1}{\mu}\left(\pi - N_0 e^{-\mu t}\right) \tag{11}$$

At $t \to \infty$ the inequality becomes $N \leq \frac{\pi}{\mu}$,which shows that the feasible solution of the system (1)-(6) as $N \to \frac{\pi}{\mu}$ is the system consist of six possible solutions and the solution model for this system is uniformly bounded in the subset of R_+^6 the feasible solution of the region **Ω** is positively invariant and attracting with respect to system(1) , the invariant region is

$$\Omega = \left\{ \left(S_E, S_U, E, C, J, R\right) \in R_+^6; N \leq \frac{\pi}{\mu} \right\} \tag{12}$$

Proposition 1: Let $R_+ > 0$, and $\rho(A)$ be positive and bounded. Then there exist $\varepsilon_0 > 0$ such that, if $0 < \varepsilon < \varepsilon_0$, then R_+ as defined in (12) is throughout Ω.

Proof: It suffices to show that for all $\mu \in \Omega$. (See Bonabeau (2002); Rodrigurz and Adrea (2010)).

Crime Free Equilibrium (CFE)

This equilibrium exist in the absence of crime related compartments E, C, J and R, that is $E = C = J = R = 0$. Thus, at equilibrium,

$$\frac{dS_E}{dt} = \frac{dS_U}{dt} = \frac{dC}{dt} = \frac{dJ}{dt} = \frac{dR}{dt} = 0 \tag{13}$$

Let E_0 be the CFE, and since at CFE, then $\pi - \mu S = 0$, implies $S = \frac{\pi}{\mu}$ and

$$E_0 = \left(\frac{\pi}{\mu}, 0, 0, 0, 0\right). \tag{14}$$

Basic Reproduction Number R_0

Applying the method of analyzing local stability of CFE in Van et al (2002), we calculate the basic reproduction number R_0 using next generation matrix operator method. The threshold quantity R_0 in epidemiological model is use for predicting disease outbreak and control strategies. Depending on the value of R_0, the disease may become persistent if $R_0 > 1$, implying that one infected individual will cause more than one secondary infection. The disease may die out in the population if $R_0 < 1$, meaning an infected individual cannot cause more than one secondary infection.

In this paper, Kalare crime is considered as transmissible disease and compute R_0 at CFE.

The associated matrix F for the new infection term is

$$F = \begin{pmatrix} 0 & \frac{\beta_1\pi_1 + \beta_2\pi_2}{\pi} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{15}$$

And

$$V = \begin{pmatrix} A & 0 & 0 & -\omega \\ -\alpha & B & \theta_4 & 0 \\ \theta_1 & \theta_2 & C & 0 \\ 0 & -\tau & \theta_3 & D \end{pmatrix}. \tag{16}$$

Now, we find the inverse of matrix V by finding,

$$\det(E_0) = |V| = ABCD - AD\theta_2\theta_4 - B\omega\theta_1\theta_3 - \omega\tau\theta_1\theta_4 - C\alpha\omega\tau - \alpha\omega\theta_2\theta_4 \tag{17}$$

$$V^{-1} = \frac{1}{|V|}\begin{pmatrix} D(B\alpha - \theta_2\theta_4) & \omega(C\tau + \theta_2\theta_4) & \omega(B\theta_2 + \tau\theta_4) & \omega(BC - \theta_2\theta_4) \\ D(C\alpha + \theta_1\theta_4) & ACD - \omega\theta_1\theta_3 & AD\theta_4 + \omega\tau\theta_3 & \omega(C\alpha + \theta_1\theta_4) \\ D(E\theta_1 + \alpha\theta_2) & AD\theta_2 + \omega\tau\theta_1 & ABD - \omega\tau\alpha & \omega((B\theta_1 + \alpha\theta_2)) \\ B\theta_1\theta_4 + C\alpha\tau + \alpha\theta_2\theta_3 + \tau\theta_1\theta_4 & A(C\tau + \theta_2\theta_3) & A(B\theta_3 + \tau\theta_4) & A(BC - \theta_2\theta_4) \end{pmatrix}. \tag{18}$$

We now find FV^{-1} and let $T = \frac{1}{\pi}(\beta_1\pi_1 + \beta_2\pi_2)$ (19)

where $A=(\mu+d_1+\alpha+\theta_1), B=(\mu+d_2+\tau+\theta_2), C=(\mu+\theta_3+\theta_4)$
$D=(\mu+\omega)$ (20)

$$FV^{-1}=\frac{1}{|V|}\begin{pmatrix} TD(C\alpha+\theta_1\theta_4) & T(ACD-\omega\theta_1\theta_3) & T(AD\theta_4+\omega\tau\theta_3) & T(\omega(C\alpha+\theta_1\theta_4)) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (21)$$

Now from $|FV^{-1}-\lambda I|$, we have

$$\lambda=0,0,0,\frac{TD(C\alpha+\theta_1\theta_4)}{|V|} \quad (22)$$

by definition $R_0=\max\{\lambda_i:\lambda_i \text{ is an eigenvalues of } FV^{-1}, i=1(1)4\}$ (23)

$$R_0=\left(\frac{\beta_1\pi_1+\beta_2\pi_2}{\pi}\right)\frac{D(C\alpha+\theta_1\theta_4)}{|V|} \quad (24)$$

Table 1. Parameter Description

Parameter	Description
π	Recruitment rate
β_1	Rate of moving from educated class into the exposed class
β_2	Rate of moving from uneducated class into the exposed class
α	Rate of moving from exposed class into the crime class
τ	Rate of moving from crime class into the recovery class
θ_1	Rate of moving from exposed class into the jail class
θ_2	Rate of moving from crime class into the jail class
θ_3	Rate of moving from jail class into the recovery class
θ_4	Rate of moving from jail class into the crime class
ω	Rate of moving from recovered class into the exposed class
μ	Natural death rate
d_1	Death induced by been exposed to the crime
d_2	Death induced by the crime

Local stability Analysis of crime free equilibrium point

The local stability analysis is obtained by trace method

Lemma1: the crime free equilibrium point is asymptotically stable if $Tr(E_0)<0, Det(E_0)>0$ whenever $R_0<1$ and unstable if $R_0>1$.

Proof:

At the E_0 we have,

$$J_{E_0} = \begin{pmatrix} -\mu & 0 & 0 & E & 0 & 0 \\ 0 & -\mu & 0 & F & 0 & 0 \\ 0 & 0 & -A & G & 0 & \omega \\ 0 & 0 & \alpha & -B & \theta_4 & 0 \\ 0 & 0 & \theta_1 & \theta_2 & -C & 0 \\ 0 & 0 & 0 & \tau & \theta_3 & -D \end{pmatrix} \quad (25)$$

where $E = \frac{\beta_1\pi_1}{\mu N}, F = \frac{\beta_2\pi_2}{\mu N}, G = \frac{\beta_1\pi_1}{\mu N} + \frac{\beta_2\pi_2}{\mu N}$ (26)

we get $Tr(E_0) = -(2\mu + A + B + C + D) < 0$ and

$$Det(E_0) = \mu^2 \begin{Bmatrix} ABCD - AD\theta_2\theta_4 - B\omega\theta_1\theta_3 - CDG\alpha - C\alpha\omega\tau \\ -DG\theta_1\theta_4 - \alpha\omega\theta_2\theta_3 - \omega\tau\theta_1\theta_4 \end{Bmatrix} \quad (27)$$

$$Det(E_0) = \mu^2 |V| \{1 - R_0\} \quad (28)$$

$$Det(E_0) = \mu^2 |V| \{1 - R_0\} > 0 \Rightarrow R_0 < 1 \quad (29)$$

hence the crime is stable.

Crime present equilibrium point

Equating the given equations to zero we have

$$S_E^* = \frac{\pi_1}{\mu + \lambda_1}, S_U^* = \frac{\pi_2}{\mu + \lambda_2} \quad (30)$$

where $\lambda_1 = \frac{\beta_1 C^*}{N}, \lambda_2 = \frac{\beta_2 C^*}{N}$ (31)

$$E^* = \frac{(\mu + \lambda_2)\lambda_1\pi_1 + (\mu + \lambda_1)\lambda_2\pi_2 + (\mu + \lambda_1)(\mu + \lambda_2)\omega R^*}{(\mu + \lambda_1)(\mu + \lambda_2)(\mu + d_1 + \alpha + \theta_1)} \quad (32)$$

$$0 = \alpha E + \theta_4 J - \mu C - \tau C - \theta_2 C - d_2 C \quad (33)$$

$$0 = \theta_1 E + \theta_2 C - \mu J - \theta_3 J - \theta_4 J \quad (34)$$

$$0 = \tau C^* + \theta_3 J^* - \mu R^* - \omega R^* \quad (35)$$

From (32) we obtain R^* and substitute into (33), and then we solve (33)-(35) simultaneously to obtain the value of E^*, C^* and J^* independently, where

$$C^* = \frac{(\theta_1\theta_4 + \alpha C) D L_1}{L_1 L_3 \left[AD(BC - \theta_2\theta_4) - \omega\theta_1(B\theta_3 + \tau\theta_4) - \alpha\omega(\tau C + \theta_2\theta_3) \right]}. \quad (36)$$

where, $L_1 = (\mu + \lambda_1)\lambda_2\pi_2 + (\mu + \lambda_2)\lambda_1\pi_1$ $L_2 = (\mu + \lambda_1)$ and $L_3 = (\mu + \lambda_2)$.

multiplying (36) by $\left(\frac{\beta_1\pi_1 + \beta_2\pi_2}{\pi} \right)$ we have,

$$\left(\frac{\beta_1\pi_1 + \beta_2\pi_2}{\pi} \right) C^* = \left(\frac{\beta_1\pi_1 + \beta_2\pi_2}{\pi} \right) \frac{(\theta_1\theta_4 + \alpha C) D L_1}{L_1 L_3 \left[AD(BC - \theta_2\theta_4) - \omega\theta_1(B\theta_3 + \tau\theta_4) - \alpha\omega(\tau C + \theta_2\theta_3) \right]} \quad (37)$$

$$\Rightarrow CPE = R_0 - \frac{L_2 L_3}{L_1} Q \quad (38)$$

$$CPE = \frac{L_2 L_3}{L_1} Q \left[\frac{L_1}{L_2 L_3 Q} R_0 - 1 \right] \quad (39)$$

Clearly the crime present equilibrium point exist and it is unique for this case if $0 < R_0 \leq \frac{L_1}{L_2 L_3 Q}$. The crime present equilibrium does not exist if otherwise.

Numerical Simulation

The time plot of figure 2 is a representation of three different scenario, with β_1 =0.9 for the first scenario, β_1 =0.6 for the second scenario and β_1 =0 for the third scenario respectively, which also depict that the number of individual population of the crime class decline from 300 to almost 170 and then stabilize for the first scenario, from 300 to almost 160 and then stabilize for the second scenario and from 300 to almost 130 and then stabilize for the third scenario respectively as time goes on.

Table 2: Parameter values

Parameter	Value	Data Source
π	1000	Estimated
μ	0.019	Estimated
β	0.66	Umar (2013)
d	0.1	Estimated
ρ	0.25	Estimated
e	0.32	Estimated
k	0, 0.2, 0.88	Umar (2013)
α	0.30	Estimated
τ	0.25	Estimated
ν	0.75	Umar (2013)
θ	0.15	Estimated

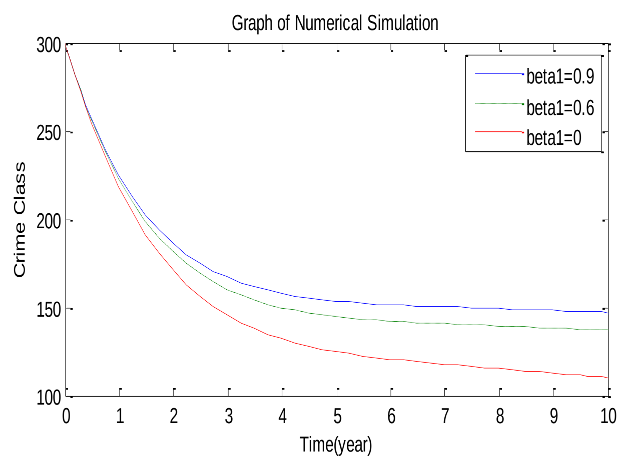

Figure 2: Graph of crime class against time.

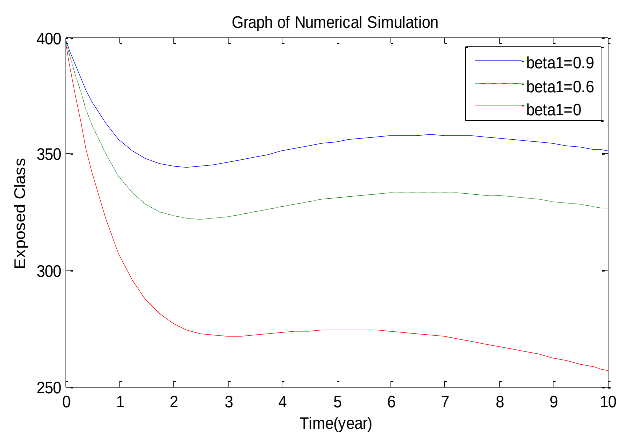

Figure 3: Graph of exposed class against time.

The time plot of figure 3 is also a representation of three different scenarios as that of figure 2 above which also depict that the number of individual population of the exposed class decline from 400 to almost 350 for the first scenario, from 400 to almost 330 for the second scenario and from 400 to almost 255 for the third scenario respectively as time goes on.

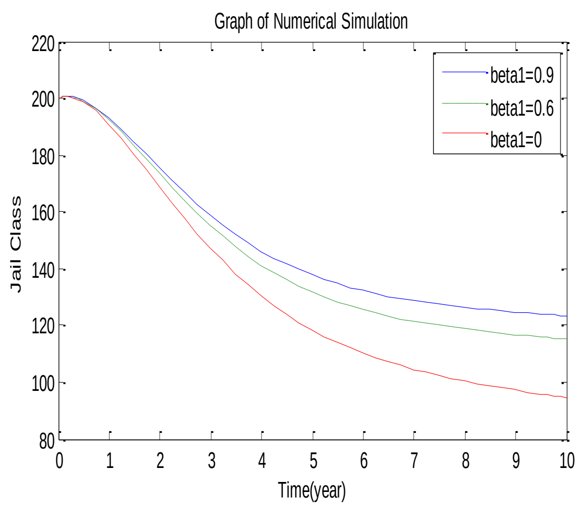

Figure 4: Graph of jail class against time.

The time plot of figure 3 is also a representation of three different scenariosas that of figure 2 and 3 above, which also depict that the number of individual population of the jail class decline from 200 to almost 150 and then stabilize for the first scenario, from 200 to almost 140 and then stabilize for the second scenario and from 200 to almost 110 and then stabilize for the third scenario respectively as time goes on.

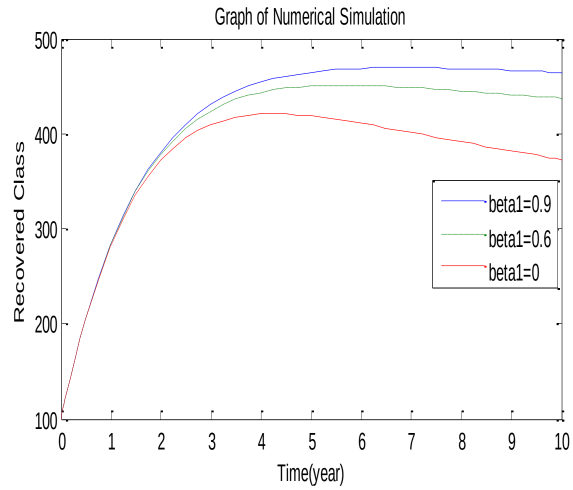

Figure 5: Graph of recovered class against time.

The time plot of figure 5 is also a representation of three different scenarios as that of figure 1 above, which also depict that the number of individual population of the Recovered class increases from 100 to almost 470 and the stabilize for the first scenario, from 100 to almost 440 and then stabilize for the second scenario and from 100 to almost 390 and then stabilize for the third scenario respectively as time goes on. This implies that, the proposed model can be said to be a good model for kalare crime control.

DISCUSSION

We have formulated a model of kalare crime in Gombe with two susceptible classes. These are the susceptible of educated and uneducated class with three different scenario as stated in the graphs above, and investigate their dynamical behavior depending on the basic reproduction number of the three different scenario, for the first scenario we have $R_0 = 0.27714 < 1$ and for the second scenario we have $R_0 = 0.20123 < 1$ and finally for the third scenario we have $R_0 = 0.0694 < 1$ which implies that all the three scenario are Kalare crime-free.

CONCLUSION

The dynamic of kalare crime was modelled into six different compartments in order to determine possible strategies for reducing and controlling crime activities in Gombe. Compartmental models have been used to study the influence of sociological and economic factors on the evolution of criminal behavior and violence in the society. The local stability analysis of crime free equilibrium point obtained using trace method and the crime present equilibrium point using **theorem1** is exists and unique for $0 < R_0 \le \frac{L_1}{L_2 L_3 Q}$, where the reproduction number $|R_0| > 0$ (i.e positive and bounded), which shows that the kalare crime can be control.

The numerical simulations of the crime class, exposed class, jail class and recovered class presented shows that the activity of kalare crime in Gombe can stabilize at each scenario as time goes on.

REFERENCES

Umar M. (2013). An Assessment of the activities of Kalare in political violence in Gombe State, Nigeria. International journal of innovative research and development, Vol. 2, pp 2278-0211.

Short M., Brantingham P., Bertozzi A., and Tita G. (2008). The stability of steady state hot-spoot Patterns for reaction diffusion models of urban crime, Math. Models, vol. 18, 1249-1267.

Nuno, J.C., Angel, M.H., and Mario, P. (2011). Mathematical Model of a Criminal Prone Society, AIMS Journals 4.1 193-207.

Sayantani, M., and Arnab, M. (2017). A mathematical model of a criminal-prone society. International journal of science and research. Vol. 6, Iss. 3; ISSN (online): 2319-7064.

Hochberg, M.E. (1991). Non-linear transmission rates and the dynamics of infectious diseases. Journal of theoretical biology. Vol. 152, no. 3, 301-321.

Bonabeau, E. (2002). Agent-based modeling: Methods and techniques for simulating human systems. Proceedings of the national academy of sciences. Vol. 99, suppl. 3, pp. 7280-7287.

Diekman, J., Hesterbeek, P., and Meetz, J. (1990). On the definition and the computation of the basic reproduction ratio R_0 in model for infectious diseases in heterogeneous population, *Mathematical Biosciences*, vol.28, pp 363-382.

Van, D., Driessche, P., and Wantmough, J. (2002). Reproduction numbers and sub-threshold endemic equilibria for compartmental models of diseases transmission, *Math. Biosci*, vol.188, pp 29-28.

Adamu, I., Bashir, S.M., Ayuba, S., and Abubakar, M.B. (2018). Mathematical assessment of social media impact on academic performance of students in higher institution. IOSR Journal of Mathematics (IOSR-JM) Vol. 14, no. 1 pp 72-79.

Feroz, S.S., Didong, L., Zhang, X., and Zhenhang, G. (2013). Mathematical modeling in criminology. Malaysian journal of mathematical sciences. Vol. 7, no. 1, pp. 125-145.

Rodriguez, N., and Andrea, B. (2010). Local existence and uniqueness of solutions to a PDE model for criminal behavior. Math. Meth. Appl. Sci. Vol. 20, pp. 1425-1457.

Robert, S.C., Chris, C., and Raul, M. (2012). Global bifurcation of solutions for crime modeling equations. Siam. J. Math. Anal. Vol. 44, pp. 1340-1358.