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A SURVEY ON THE SOLVABILITYAND SIMPLICITY STATUS OF PERMUTATION GROUPS

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ABSTRACT

This paper is a survey for the solvability and simplicity status of a permutation group G whose order is of the form rp^r, where r is any prime and p is odd prime. The group of this form can be p-group or any group generated by wreath products of two permutation groups depending on the value of r and p. The concept of p groups and wreath products of two permutation groups has been applied to explore the groups of interest and later investigation has been carried out to test them for the above stated status.

Keywords:

INTRODUCTION

The study as we know is focused on the solvability and simplicity status of permutation groups. This concept of solvability and simplicity statuses are very important in the theory of permutation groups. The two concepts go hand in hand with abelian groups which are also very important concepts of the theory of groups entirely. That is why the attention of mathematicians focused on these two statuses.

In order to understand the study so many definitions are introduced, introducing also is the famous Sylow's Theorem. This is because for a group theorist as someone put it and we quote "Sylow's Theorem is such a basic tool, and so fundamental that it is used almost without thinking, like breathing".

Definition

The series of subgroups $G_0, G_1, G_2, \dots, G_n$ such that $G = G_n \supset G_{n-1} \supset G_{n-2} \supset \cdots \supset G_1 \supset G_0 = \{1\}$ where G_i/G_{i+1} is abelian, is called a solvable series.

Definition 1.2 (Milne, J.S, 2009)

A group G is solvable if there is a finite collection of groups $G_0 G_1 \dots\ G_n$ such that$(1) = G_0 \subseteq G_1 \subseteq \cdots \subseteq G_n = G$ where $G_i \trianglelefteq G$ and $G_{i+1}/G_i$is abelian.If $|G| = 1$ then G is considered as solvable group.

Definition A finite group is simple when its only normal subgroups are the trivial subgroup and the whole group.

Since the study is majorly on group and its properties, we pause and state one of the fundamental theorems on group, i.e., Sylow's Theorem.

MATERIALS AND METHODS

Sylow's Theorem 1.1 (Audu, 2003)

Let G be a group of order $p^{\alpha}m$, where p is a prime, $m \geq 1$, and p does not divide m. Then:

I. $Syl_p(G) \neq \emptyset$, i.e., Sylow p-subgroups exists.

II. All Sylow p-subgroups are conjugate in G, i.e., if p_1 and p_2 are both Sylow p-subgroups, then there is some $g \in G$ such that $p_1 = gp_2g^{-1}$. In particular, $n_p(G) = (G: N_G(P))$.

III. Any p-subgroup of G is contained in a Sylow p-subgroup.

IV. $n_p(G) \equiv 1 \ mod\ p$.

WREATH PRODUCT (Audu, 2003)

The Wreath product of C by D denoted by W = C wr D is the semi-direct product of P by D so that$W = \{(fd) \mid f \in Pd \in D\}$ with multiplication in W defined as $(f_1d_1)(f_2d_2) = f_1{f_2}^{{d_1}^{-1}}(d_1d_2)$ for all $f_1f_2 \in Pandd_1d_2 \in D$. Henceforth we write f d instead of (fd) for elements of W.

Theorem 1.2 (Audu, 2003)

Let D act on P as $f^d(\delta) = f(\delta d^{-1})$ where $f \in Pd \in Dand\delta \in \Delta$. Let W be the group of all juxtaposed symbols f d with $f \in Pd \in D$ and multiplication given by $(f_1d_1)(f_2d_2) = f_1{f_2}^{{d_1}^{-1}}(d_1d_2)$. Then W is a group called the semi-direct product of P by D with the defined action.

Based on the forgoing we note the following:

- If C and D are finite groups then the wreath product W determined by an action of D on a finite set is a finite group of order $|W| = |C|^{|\Delta|}.|D|$.
- P is a normal subgroup of W and D is a subgroup of W.
- The action of W on $\Gamma \times \Delta$ is given by $(\alpha\beta)fd = (\alpha f(\beta)\beta d) where \alpha \in \Gamma and \beta \in \Delta$.

We shall at this point identify the conditions under which a subgroup will be soluble or nilpotent and study them for further investigation.

Theorem 1.3 (Thanos, 2006)

G is solvable if and only if $G^{(n)} = 1$, for some n.

Proposition

Let G be solvable and $H \leq G$. Then

1. H is solvable.
2. If $H \lhd G$, then G/H is solvable.

Proof

Start from a series with abelian slices. $G = G_0 \trianglerighteq G_1 \trianglerighteq \cdots \trianglerighteq G_n = \{1\}$. Then $H = H \cap G_0 \trianglerighteq H \cap G_1 \trianglerighteq \cdots \trianglerighteq H \cap G_n = \{1\}$. When H is normal, we use the canonical projection $\pi : G \to G/H$ to get $G/H = \pi(G_0) \trianglerighteq \cdots \trianglerighteq \pi(G_n) = \{1\}$; the quotients are abelian as well, so G/H is still solvable.

Theorem 1.5 (Milne, 2009)

A group G is solvable if and only if it has a solvable series.

Thus; we give the following illustrations:

(i) $G = \{(1), (48765), (47586), (46857), (45678), (132)$
$(132)(48765), (132)(47586), (132)(46857), (132)(45678), (123)$
$(123)(48765), (123)(47586), (123)(46857), (123)(45678)\}$

has the subgroups as follows;

$$H_0 = (1)$$

$$H_1 = \{(1)(123)(132)\}$$

$$H_2 = \{(1)(48765)(47586)(46857)(45678)\}$$

$H_3 = \{(1)(48765)(47586)(46857)(45678), (132)$
$(132)(48765), (132)(47586), (132)(46857), (132)(45678), (123)$
$(123)(48765), (123)(47586), (123)(46857), (123)(45678)\}$

has a solvable series which is $(1) = H_0 \lhd H_1 \lhd H_3 = G$ hence solvable by Theorem 2.3

(ii) The dihedral group D_n is solvable since $D_n \rhd \langle p \rangle \rhd \{1\}$
Let D_{16} be the Dihedral group of Degree 8 given by:

$D_{16} = \{(1)(28)(37)(46)(15)(26)(37)(48)(15)(24)(68)(1753), (2864)$
$(17)(26)(35)(1357)(2468)(13)(48)(57)(18765432),$
$(18)(27)(36)(45)(14725836), (14)(23), (58)(67), (16385274)(16)(25)$
$(34)(78), (12345678), (12), (38), (47), (56)\}$

whose subgroups are as follows;

$$H_1 = (1)$$

$$H_2 = \{(1)(15)(26)(37)(48)\} = \langle p \rangle$$

$H_3 = \{(1)(28)(37)(46)(15)(26)(37)(48)(15)(24)(68)(1753)(2864)$
$(17)(26)(35)(1357)(2468)(13)(48)(57)(18765432)$
$(18)(27)(36)(45)(14725836)(14)(23)(58)(67)(16385274)(16)(25)$
$(34)(78)(12345678)(12)(38), (47), (56)\}$

Hence $D_{16} = H_3 \rhd H_2 \rhd H_1 = (1)$

Proposition 1.6

A group G is a direct product of subgroups $H_1,\ H_2$ if and only if

a) $G = H_1 H_2$
b) $H_1 \cap H_2 = \{e\}$ and
c) Every element of H_1 commutes with every element of H_2.

Proof

If G is the direct product of $H_1\ and\ H_2$, then certainly (a) and (c) hold and (b) holds because, for any $g \in H_1 \cap H_2$, the element (g, g^{-1})maps to e under $(h_1, h_2) \mapsto h_1 h_2$ and so equals to (e, e).

Conversely, (c) implies that $(h_1, h_2) \mapsto h_1 h_2$ is a homomorphism and (b) implies that it is injective:

$h_1 h_2 = e \Longrightarrow h_1 = {h_2}^{-1} \in H_1 \cap H_2 = \{e\}$. Finally, (a) implies that it is surjective.

Proposition 1.7

A group G is a direct product of subgroups $H_1,\ H_2$ if and only if

a) $G = H_1 H_2$
b) $H_1 \cap H_2 = \{e\}$
c) $H_1\ and\ H_2$ are both normal in G

Proof

Certainly, these conditions are implied by those in proposition 1.5 and so it remains to show that they imply that each element h_1 of H_1 commutes with each element h_2 of H_2 .

Two elements h_1 and h_2 of G commute if and only if their commutator $[h_1, h_2] \stackrel{def}{=} (h_1 h_2)(h_2 h_1)^{-1}$ i.e. e but

$$(h_1 h_2)(h_2 h_1)^{-1} = h_1 h_2 {h_1}^{-1} {h_2}^{-1}$$
$$= \begin{cases} (h_1 h_2 {h_1}^{-1}).\ {h_2}^{-1} \\ h_1.(h_2 {h_1}^{-1} {h_2}^{-1}) \end{cases}$$

Which is in H_2 because H_2 is normal and is in H_1 because H_1 is normal. Therefore (b) implies that $[h_1, h_2] = e$.

Proposition 1.8

Any group of order $ppnn$ where p is a prime, is solvable.

Proof

We prove the proposition by induction on nn. For nn = 0, the proposition is trivial. Let n ≥ 1 and assume that the proposition is true for rr<nn. Let G be a group of order $ppnn$. Then by a Proposition, the centre C of G has order $ppss$ where s ≥ 1. Then the order of GG/CC is $ppnn{-}ss$and $nn - ss$<nn. By the induction hypothesis G/C is solvable. Milne (2013).

RESULTS

Theorem 1.9

Let G be a group of order rp^r where p is an odd prime and k is an integer greater than or equals to 2 (and that p and q are not consecutive in any case), then

a) G is not simple.
b) G is Solvable

Proof

a) It is clear by second Sylow theorem that G has Sylow r-subgroups and Sylow p-sub groups. By second Sylow theorem n_r *could be* $1, p, p^2, \ldots, p^r$ *and* $n_p = 1$, implying that Syl_p is normal showing that G is not simple as required.

b) Suppose G is a group of order rp^r, clearly G has Sylow r-subgroups and Sylow p-subgroups for all odd prime p. Let N be the Sylow r-subgroup of G then N is unique (one) by Sylow second theorem and hence normal in G of prime order and solvable by Proposition 1.8. Being N a normal subgroup of G, it forms a factor group G/N of order p^r which is a p-group and this subgroup is solvable by Proposition 1.8. it follows that G is solvable by Proposition 1.7

APPLICATION

Consider the permutation groups $C_1$and D_1

$C_1 = \{(1), (12)\},\ D_1 = \{(1), (3{,}4{,}5), (3{,}5{,}4)\}$acting on the sets $S_1 = \{1{,}2\}$ *and* Δ_1= {3,4,5} respectively.

Let $P = {C_1}^{\Delta_1} = \{f: \Delta_1 \longrightarrow C_1\}$ *then* $|P| = |C_1|^{\Delta_1} = 2^3 = 8$

We can easily verify that $W_1$is a group with respect to the operations

$(f_1 f_2)\delta_1 = f_1(\delta_1) f_2(\delta_1)$ *where* $\delta_1 \in \Delta_1$.

The wreath product of C_1 and D_1 is given by W_1 as follows:

```
gap> C1:=Group((1,2));
Group([ (1,2) ])
```

gap> D1:=Group((3,4,5));
Group([(3,4,5)])
gap> W1:=WreathProduct(C1,D1);
Group([(1,2), (3,4), (5,6), (1,3,5)(2,4,6)])
gap> Order(W1);
24
gap> IsSimple(w1);
false
gap> IsNilpotent(W1);
false
gap> quit;

Consider the permutation groups $C_2$and D_2

$$C_2 = \{(1),(12)\},$$

D_2
$= \{(1),(37654),(36475),(35746),(34567)\}$
acting on the sets $S_2 = \{1,2\} and \Delta_2 = \{3,4,5,6,7\}$ respectively.
Let $P = {C_2}^{\Delta_2} = \{f: \Delta_2 \longrightarrow C_2\} then |4P| = |C_2|^{\Delta_2} = 2^5 = 32$

We can easily verify that $W_2$is a group with respect to the operations

$(f_1 f_2)\delta_1 = f_1(\delta_1) f_2(\delta_1) where \delta_1 \in \Delta_1$.

The wreath product of C_2 and D_2 is given by W_2 as follows:
gap> C2:=Group((1,2));
Group ([(1,2)])
gap> D2:=Group((3,4,5,6,7));
Group ([(3,4,5,6,7)])
gap> W2:=WreathProduct(C2,D2);
Group ([(1,2), (3,4), (5,6), (7,8), (9,10), (1,3,5,7,9)(2,4,6,8,10)])
gap> Order(W2);
160
gap> IsSolvable(W2);
true
gap> IsSimple(w2);
false
gap> quit;

Consider the permutation groups $C_3$and D_3

$$C_3 = \{(1),(12)\},$$

D_3
$= \{(1),(\ 113121110\ 9\ 8\ 7\ 6\ 5\ 4\ 3\ 2),(\ 11210\ 8\ 6\ 4\ 21311\ 9\ 7\ 5\ 3),$
$(\ 111\ 8\ 5\ 212\ 9\ 6\ 31310\ \ 7\ 4),(\ 110\ 6\ 211\ 7\ 312\ 8\ 413\ 9\ 5),(\ 1\ 9\ 412\ 7\ 210\ 513\ 8\ 311\ 6),$
$(\ 1\ 8\ 2\ 9\ 310\ 411\ 512\ 613\ 7),(\ 1\ 713\ 612\ 511\ 410\ 3\ 9\ 2\ 8),(\ 1\ 611\ 3\ 813\ 510\ 2\ 712\ 4\ 9),$
$(\ 1\ 5\ 913\ 4\ 812\ 3\ 7\ 11\ 2\ 610),(\ 1\ 4\ 71013\ 3\ 6\ 912\ 2\ 5\ 811),(\ 1\ 3\ 5\ 7\ 91113\ 2\ 4\ 6\ 81012),$
$(\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 910111213)\}$
acting on the sets $S_3 = \{1,2\} and \Delta_3 = \{1,2,3,4,5,6,7,8,9,10,11,12,13\}$ respectively.
Let $P = {C_3}^{\Delta_3} = \{f: \Delta_3 \longrightarrow C_3\} then |P| = |C_3|^{\Delta_3} = 2^{13} = 8192$

We can easily verify that$W_3$is a group with respect to the operations

$(f_1 f_2)\delta_1 = f_1(\delta_1) f_2(\delta_1) where \delta_1 \in \Delta_1$.

The wreath product of C_3 and D_3 is given by W_3 as follows:

gap> C3:=Group ((1,2));
Group ([(1,2)])
gap>
D3:=Group((3,4,5,6,7,8,9,10,11,12,13,14,15
));
Group ([(3,4,5,6,7,8,9,10,11,12,13,14,15)])
gap> W3: =WreathProduct(C3, D3);
Group ([(1,2), (1,3,5,7,9,11,13,15,17,19,21,23,25)(2,4,6,8,10,12,14,16,18,20,22,24,26)])
gap> Order(W3);
106496
gap> IsSolvable(W3);
true
gap> IsSimple(w3);
false
gap> quit;

Consider the permutation groups C_4 and D_4

$$C_4 = \{(1), (123), (132)\},$$

$$D_4 = \{(1), (48765), (47586), (46857), (45678)\}$$

acting on the sets $S_4 = \{1,2,3\} and \Delta_4 = \{4,5,6,7,8\}$ respectively.

Let $P = {C_4}^{\Delta_4} = \{f: \Delta_4 \longrightarrow C_4\} then |P| = |C_4|^{\Delta_4} = 3^5 = 243$

We can easily verify that $W_4$is a group with respect to the operations

$$(f_1 f_2)\delta_1 = f_1(\delta_1) f_2(\delta_1) where \delta_1 \in \Delta_1 .$$

The wreath product of C_4 and D_4 is given by W_4 as follows:

```
gap> C4:=Group((1,2,3));
Group([ (1,2,3) ])
gap> D4:=Group((4,5,6,7,8));
Group([ (4,5,6,7,8) ])
gap> W3:=WreathProduct(C4,D4);
Group([ (1,2,3), (4,5,6), (7,8,9), (10,11,12),
(13,14,15),
(1,4,7,10,13)(2,5,8,11,14)(3,6,9,12,15) ])
gap> Order(W4);
1215
gap> IsSolvable(W4);
true
gap> IsSimple(w4);
false
gap> quit;
```

Consider the permutation groups $C_5$and D_5

$$C_5 = \{(1), (123), (132)\},$$

$$D_5 = \{(1), (\ 410\ 9\ 8\ 7\ 6\ 5), (\ 4\ 9\ 7\ 510\ 8\ 6), (\ 4\ 8\ 5\ 9\ 610\ 7), (\ 4\ 710\ 6\ 9\ 5\ 8), (\ 4\ 6\ 810\ 5\ 7\ 9), (\ 4\ 5\ 6\ 7\ 8\ 910)\}$$

acting on the sets $S_5 = \{1,2,3\} and \Delta_5 = \{4,5,6,7,8,9,10\}$ respectively.

Let $P = {C_5}^{\Delta_5} = \{f: \Delta_5 \longrightarrow C_5\} then |P| = |C_5|^{\Delta_5} = 3^7 = 2187$

We can easily verify that $W_5$is a group with respect to the operations

$$(f_1 f_2)\delta_1 = f_1(\delta_1) f_2(\delta_1) where \delta_1 \in \Delta_1 .$$

The wreath product of C_5 and D_5 is given by W_5 as follows:

```
gap> C5:=Group((1,2,3));
Group ([ (1,2,3) ])
gap> D5: =Group((4,5,6,7,8,9,10));
Group ([ (4,5,6,7,8,9,10) ])
gap> W5:=WreathProduct(C5,D5);
<permutation group of size 15309 with 8
generators>
gap> Order(W5);
15309
gap> IsSolvable(W5);
true
gap> IsSimple(w5);
false
gap> quit;
```

Consider the permutation groups $C_6$and D_6

$$C_6 = \{(1), (\ 11110\ 9\ 8\ 7\ 6\ 5\ 4\ 3\ 2), (\ 110\ 8\ 6\ 4\ 211\ 9\ 7\ 5\ 3), (\ 1\ 9\ 6\ 311\ 8\ 5\ 210\ 7\ 4), (\ 1\ 8\ 411\ 7\ 310\ 6\ 2\ 9\ 5), (\ 1\ 7\ 2\ 8\ 3\ 9\ 410\ 511\ 6), (\ 1\ 611\ 510\ 4\ 9\ 3\ 8\ 2\ 7), (\ 1\ 5\ 9\ 2\ 6\ 10\ 3\ 711\ 4\ 8), (\ 1\ 4\ 710\ 2\ 5\ 811\ 3\ 6\ 9), (\ 1\ 3\ 5\ 7\ 911\ 2\ 4\ 6\ 810), (\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 910\ 11)\},$$

$$D_6 = \{(1), (12\ 13)\}$$

acting on the sets $S_6 = \{1,2,3,4,5,6,7,8,9,10,11\} and \Delta_6 = \{12,13\}$ respectively.

Let $P = {C_6}^{\Delta_6} = \{f: \Delta_6 \longrightarrow C_6\} then |P| = |C_6|^{\Delta_6} = 11^2 = 121$

We can easily verify that $G_1$is a group with respect to the operations

$(f_1 f_2)\delta_1 = f_1(\delta_1) f_2(\delta_1) where \delta_1 \in \Delta_1$.

The wreath product of C_2 and D_2 is given by W_2 as follows:

```
gap> C6:=Group((1,2,3,4,5,6,7,8,9,10,11));
Group([ (1,2,3,4,5,6,7,8,9,10,11) ])
gap> D6:=Group((12,13));
Group([ (12,13) ])
gap> W6:=WreathProduct(C6,D6);
Group([(1,2,3,4,5,6,7,8,9,10,11),
(12,13,14,15,16,17,18,19,20,21,22),
(1,12)(2,13)(3,14)(4,15)(5,16)(6,17)(7,18)
(8,19)(9,20)(10,21)(11,22) ])
gap> Order(W6);
242
gap> IsSolvable(W6);
true
gap> IsSimple(w6);
false
```

Consider the permutation groups C_7 and D_7

$C_7 = \{(1),(\ 1\ 11\ 10\ 9\ 8\ 7\ 6\ 5\ 4\ 3\ 2),(\ 1\ 10\ 8\ 6\ 4\ 2\ 11\ 9\ 7\ 5\ 3),(\ 1\ 9\ 6\ 3\ 11\ 8\ 5\ 2\ 10\ 7\ 4),$
$(\ 1\ 8\ 4\ 11\ 7\ 3\ 10\ 6\ 2\ 9\ 5),(\ 1\ 7\ 2\ 8\ 3\ 9\ 4\ 10\ 5\ 11\ 6),(\ 1\ 6\ 11\ 5\ 10\ 4\ 9\ 3\ 8\ 2\ 7),$
$(\ 1\ 5\ 9\ 2\ 6\ 10\ 3\ 7\ 11\ 4\ 8),(\ 1\ 4\ 7\ 10\ 2\ 5\ 8\ 11\ 3\ 6\ 9),(\ 1\ 3\ 5\ 7\ 9\ 11\ 2\ 4\ 6\ 8\ 10),$
$(\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11)\},$

$D_7 = \{(1),(12\ 16\ 15\ 14\ 13),(12\ 15\ 13\ 16\ 14),(12\ 14\ 16\ 13\ 15),(12\ 13\ 14\ 15\ 16)\}$

acting on the sets $S_7 = \{1,2,3,4,5,6,7,8,9,10,11\} and \Delta_7 = \{12,13,14,15,16\}$ respectively.

Let $P = {C_7}^{\Delta_7} = \{f: \Delta_7 \longrightarrow C_7\} then |P| = |C_7|^{\Delta_7} = 11^5 = 161{,}051$

We can easily verify that $W_7$is a group with respect to the operations

$(f_1 f_2)\delta_1 = f_1(\delta_1) f_2(\delta_1) where \delta_1 \in \Delta_1$.

The wreath product of C_7 and D_7 is given by W_7 as follows:

```
gap> C7:=Group((1,2,3,4,5,6,7,8,9,10,11));
Group ([ (1,2,3,4,5,6,7,8,9,10,11) ])
gap> D7:=Group((12,13,14,15,16));
Group ([ (12,13,14,15,16) ])
gap> Order(W7);
gap> W7:=WreathProduct(C7,D7);
<permutation group of size 805255 with 6 generators>
gap> Order(W7);
805255
gap> IsSolvable(W7);
true
gap> IsSimple(w7);
false
gap> quit;
```

Consider the permutation groups C_8 and D_8

$C_8 = \{(1),(\ 1\ 13\ 12\ 11\ 10\ 9\ 8\ 7\ 6\ 5\ 4\ 3\ 2),(\ 1\ 12\ 10\ 8\ 6\ 4\ 2\ 13\ 11\ 9\ 7\ 5\ 3),(\ 1\ 11\ 8\ 5\ 2\ 12\ 9\ 6\ 3\ 13\ 10\ 7\ 4),(\ 1\ 10\ 6\ 2\ 11\ 7\ 3\ 12\ 8\ 4\ 13\ 9\ 5),(\ 1\ 9\ 4\ 12\ 7\ 2\ 10\ 5\ 13\ 8\ 3\ 11\ 6),(\ 1\ 8\ 2\ 9\ 3\ 10\ 4\ 11\ 5\ 12\ 6\ 13\ 7),(\ 1\ 7\ 13\ 6\ 12\ 5\ 11\ 4\ 10\ 3\ 9\ 2\ 8),(\ 1\ 6\ 11\ 3\ 8\ 13\ 5\ 10\ 2\ 7\ 12\ 4\ 9),(\ 1\ 5\ 9\ 13\ 4\ 8\ 12\ 3\ 7\ 11\ 2\ 6\ 10),(\ 1\ 4\ 7\ 10\ 13\ 3\ 6\ 9\ 12\ 2\ 5\ 8\ 11),(\ 1\ 3\ 5\ 7\ 9\ 11\ 13\ 2\ 4\ 6\ 8\ 10\ 12),(\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13)\}$

$D_8 = \{(1),(14\ 18\ 17\ 16\ 15),(14\ 17\ 15\ 18\ 16),(14\ 16\ 18\ 15\ 17),(14\ 15\ 16\ 17\ 18)\}$

acting on the sets $S_8 = \{1,2,3,4,5,6,7,8,9,10,11\} and \Delta_8 = \{12,13,14,15,16\}$ respectively.

Let $P = {C_8}^{\Delta_8} = \{f: \Delta_8 \longrightarrow C_8\} then |P| = |C_8|^{\Delta_8} = 13^5 = 371{,}293$

We can easily verify that $W_7$is a group with respect to the operations

$(f_1 f_2)\delta_1 = f_1(\delta_1)f_2(\delta_1) where \delta_1 \in \Delta_1$.

The wreath product of C_7 and D_7 is given by W_7 as follows:

```
gap>
C8:=Group((1,2,3,4,5,6,7,8,9,10,11,12,13));
Group([ (1,2,3,4,5,6,7,8,9,10,11,12,13) ])
gap> D8:=Group((14,15,16,17,18));
Group([ (14,15,16,17,18) ])
gap> W8:=WreathProduct(C8,D8);
<permutation group of size 1856465 with 6
generators>
gap> Order(W8);
1856465
gap> IsSolvable(W8);
true
gap> IsSimple(w8);
false
gap> quit;
```

DISCUSSION

It can be seen from the result that any group G whose order is rp^r for any prime r and odd prime p;

a) Is solvable for all values of p and r.
b) Is not simple.

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