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TIME SERIES MODELING OF CURRENCIES IN CIRCULATION IN NIGERIA USING ARIMA MODEL

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Abstract

Currency in circulation is the outstanding amount for notes and coins circulated in the economy and they are the most liquid monetary aggregate. Currency in circulation (CIC) accounts for approximately seventy percent of reserve money in Nigeria. This research is conducted to model and forecast currency in circulation in Nigeria from 2006 to 2015 with different types of ARIMA family models. ARIMA (1, 1, 0) served as the best model among the 12 postulated ARIMA models because it has the least values of Akaike information criteria (AIC) and Bayesian information criteria (BIC). And also the research proposed the ARIMA model for currency in circulation (CIC), the model evaluate the performance and forecast three years observation (i.e from 2016 to 2018) of the process and the results shows an increase of currency in circulation in Nigeria from 2016 to 2018.

Key Words: Currency in Circulation, ARIMA, Model Evaluation Criteria, Nigeria

Introduction

Currency in circulation (CIC) refers to notes and coins held outside banks and are the most liquid monetary aggregate. Currency in circulation, together with demand deposits is a component of narrow money movements, of which are of interest to policy makers. Currencies in circulation (CIC) dynamics are often considered as an indicator for monetization or demonetization of the economy. Two most relevant indicators showing the relative significance of currency in circulation(CIC) in any economy are (1) share of currency in circulation(CIC) in money supply and (2) ratio of currency in circulation (CIC) to nominal gross domestic product (Stavreski, 1998). An increase (decrease) in currency demand reduces (increases) the availability of liquidity. In the short-run, the demand for currency is mainly affected by seasonal factors or exceptional events (such as retrospective pay increases), the patterns of which could be identified from historical data. Separate forecasts could also be obtained from the banking sector to improve the central bank's forecasts. In the long run, the demand for currency depends upon macroeconomic variables (e.g., GDP, interest rates etc.), and such forecasts could possibly facilitate shifts in the demand function over time (Stavreski, 1998). To model and forecast currency in circulation, generally two types of approaches are followed. The first approach is through a standard currency demand equation based on the theory of transaction and portfolio demand for money. Such an equation could be estimated in isolation (Jadhav, 1994). Alvan (2014) Modeled and forecasted currency in circulation in Nigeria for liquidity management using VAR and VEC models and observed that exchange rate, bank rate, seasonality, holiday and election were significant in explaining demand for currency, Tillers Ivars (2002) compared the forecasting performance of typical linear forecasting models, namely the regression model and the seasonal ARIMA model using daily data, found that seasonal ARIMA model performs better in forecasting CIC, particularly for short-term horizons. Autoregressive Integrated Moving Average (ARIMA) models intend to describe the current behaviour of variables in terms of linear relationships with their past values. These models are also called Box-Jenkins (1976) models on the basis of these authors" pioneering work regarding time-series forecasting techniques. An ARIMA model can be decomposed in two parts. First, it has an Integrated (I) component (d), which represents the amount of differencing to be performed on the series to make it stationary. The second component of an ARIMA consists of an ARMA model for the series rendered stationary through differentiation. The ARMA component is further decomposed into AR and MA components. The autoregressive (AR) component captures the correlation between the current value of the time series and some of its past values. For example, AR(1) means that the current observation is correlated with its immediate past value at time *t-1*. The Moving Average (MA) component represents the duration of the influence of a random (unexplained) shock. For example, MA (1) means that a shock on the value of the series at time *t* is correlated with the shock at *t-1*. ARIMA models produce accurate forecasts based on the historical patterns of the time series data. ARIMA belongs to the class of linear models and can represent both stationary and non-stationary data. ARIMA models do not involve the dependent variable; instead they make use of information in the series to generate the series itself. Stationary series is the one which vary about a fixed value and non-stationary series do not vary about a fixed value (Box-Jenkins 1976).

Methodology

Autoregressive (AR) Process

Let Y_t represent a time series, then

$$(Y_t-\delta) = \alpha_1 (Y_{t-1}-\delta) + \varepsilon_t \qquad (1)$$

Where δ is the mean of Y and ε_t which is an un correlated random error term with mean zero and constant variance σ^2 (it's a white noise) then we say that Y_t follows a first order autoregressive, or AR(1), stochastic process. Here the value of Y at time t depend on its value in the previous time period and random term but if we consider this model

$$(Y_t-\delta) = \alpha_1 (Y_{t-1}-\delta) + \alpha_2 (Y_{t-2}-\delta) + \varepsilon_t \qquad (2)$$

Then we say that Y_t follows a second-order autoregressive AR (2) process. That is the value of Y at time t depends on its value in the previous two time periods, where the Y value are being expressed around their mean value δ. In general, we can have

$$(Y_t-\delta) = \alpha_1 (Y_{t-1}-\delta) + \alpha_2 (Y_{t-2}-\delta) + \ldots + \alpha_p (Y_{t-p}-\delta) + \varepsilon_t \qquad (3)$$

In which case Y_t is a p^{th}-order, or AR (p) process.

Equation (3) can also be written as

$Y_t = \tau + \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + \ldots + \alpha_p Y_{t-p} + \varepsilon_t$

Where $\tau = (1-\sum_i^p \alpha_i)\,\delta$

Therefore, an AR model is simply a linear regression of the current value of the series against one more prior value of the series. The value of P is called the order of the AR model. The model can be analyzed with one of various methods, including standard linear least square (OLS) techniques.

Moving Avarge (MA) Model

Another common approach for modeling univariate time series data is the MA model. Suppose we model Y_t as follows

$Y_t = \varnothing + \beta_o \varepsilon_t + \beta_1 \varepsilon_{t-1}$ (4)

Where $\varnothing$ is a constant and ε_t, is the random error shock. Here Y_t is equal to a constant plus a moving averages of the current and past error term. Thus, we say that Y_t follows a first-order moving averages, or an MA (1) process. If Y_t is model as

$Y_t = \varnothing + \beta_o \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2}$

then it is an MA (2) process. More generally

$Y_t = \varnothing + \beta_o \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \ldots + \beta_p \varepsilon_{t-p}$ (5)

Then Y_t is an MA (p) (i.e moving averages of order p). Therefore an MA process is simply a linear combination of white noise error terms.

Autoregressive and Moving Avarages (ARMA) Process

It is quite likely that Y_t has characteristics of both AR and MA process and is therefore ARMA. Thus, Y_t follows an ARMA (1, 1) process if it can be written as

$Y_t = w + \alpha_1 Y_{t-1} + \beta_o \varepsilon_t + \beta_1 \varepsilon_{t-1}$ (6)

That is Y_t is linear of one AR and one MA term, where w is a constant term. In general, in an ARMA (p,q) process; there will be p autoregressive and q moving averages terms.

ARIMA Model

The Univariate Autoregressive Moving Average (ARMA) model is a time series medel that uses past and current values of the dependent variable to produce forecasts of the variable. The technique generates that this identified correlation will continue into the future. In this way, it becomes possible to obtain good approximation of the behavior of a variable by a purely statistical approach. Based on the Box-Jenkins (1976) modeling technique, ARMA methodology seeks to establish a parsimonious relationship, using as few parameters as possible. For example to forecast the values of a series y, using the ARMA technique, the general model specification for the series is expressed a

$y_t = (a_1L + a_2L^2 + \ldots + a_pL^P)\, y_t + (1 + b_1L + \ldots + b_q L^q)\, e_t$ (7)

Which can be expressed as

$$y_t = \sum_{i=1}^{p} (a_i L^p) y_t + \sum_{i}^{q} (b_i L^q) e_t + e_t$$

Where P and q = the number of lags for autoregressive (AR) and moving average (MA) processes respectively;
e_t = an error process, with $e_t \sim N(0,1)$

L =the lag operator on the processes; defined as $L^n y_t = y_{t-n}$ or $L y_t = y_{t-1}$

The specification can be further extended to include explicit modelling of seasonal factors observed in the data. Apart from specifying seasonal dummy variables, the pure time series ARMA Specification is extended to the Seasonal Autoregressive Moving Average (SARMA). Detail definition of ARIMA models are stated as follows. To determine the appropriate lag lengths of the processes, examination of the autocorrelation (ACF) and Partial autocorrelation (PACF) functions is necessary, as these functions give the relationship between data points, and indicates the memory of the data generation process.
An ARIMA-seasonal model is denoted ARIMA(P,D,Q) , where P is the order of auto regression in the seasonal model, D is the order of differencing, Q is the order of the moving average in the seasonal model and S is the seasonal length.
A seasonal-ARIMA (P, D, Q) *S* model is given by

$$(1-\beta_1-\ldots-\beta_p L^{sp})(1-L^s)^D y_t = (1-\Phi_1 L^s-\ldots-\Phi_Q L^{QS})\,\varepsilon_t \qquad (8)$$

Statistical Test

Augmented Dickey Fuller (ADF) Test

This test was first introduced by Dickey and fuller (1979) to test for the presence of unit root (s)

$$\Delta y = \alpha y_{t-1} + \beta_1 \Delta y_{t-1} + \beta_2 \Delta y_{t-2} + \ldots + \beta_p \Delta y_{t-p} \qquad (9)$$

The hypothesis testing

Ho: $\alpha = 0$ (the series contains unit roots)

H_1: $\alpha < 0$ (the series is stationary)

Decision Rule:

Reject the null hypothesis (Ho) if the test statistic is less than the asymptotic critical values.

KPSS Test (Kwiatkowski Phillips Schmidt Shin Test)

This test is used to test for stationary in level (i.e. mean) by considering

$$Y_t = X_t + Z_t \qquad (10)$$

The integration properties of a series Y_t may be investigated by testing:
Ho: $Y_t \sim t$
H1: $Y_t \sim t$
That is, the null hypothesis that the data generating process (DGP) is stationary is tested against a unit root and KPSS.

Model Selection Methods

The most famous Information Criteria are Akaike Information Criteria (AIC), the Bayesian Swartz Information Criteria (SIC) are considered for model selection. These criteria are computed using the log-likelihood estimates. Given the criteria

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values of two or more models, the model satisfying minimum AIC or SIC is most representatives of the true model and, may be interpreted as the best approximating model among those being considered (Dayton 2003, Hamadu and Adeleke, 2009). Let r, k, n and *ll* be response variable, the number parameters, the number of observations and the maximum likelihood function respectively. The Akaike Information Criteria is

$$AIC = -2\left(\frac{ll}{n}\right) + \frac{2k}{n} \quad (11)$$

The Schwartz Bayesian information criteria is an alternative the AIC that imposes a large penalty for additional coefficients. It is given as:

$$SIC = -2\left(\frac{ll}{n}\right) + \frac{k \ln n}{n} \quad (12)$$

The main reason for preferring the use of a model selection procedure such as SIC in comparison to traditional significance tests is the fact that, a single holistic decision can be made concerning the model that is best supported by the data in contrast to what is usually a series of possibly conflicting significance test. Moreover, models can be ranked from best to worst supported by the data at hand, thus, enlargeing the possibilities of interpretation (for more insights see Dayton, 2003, Hamadu and Ibiwoye, 2010).

Empirical Analysis

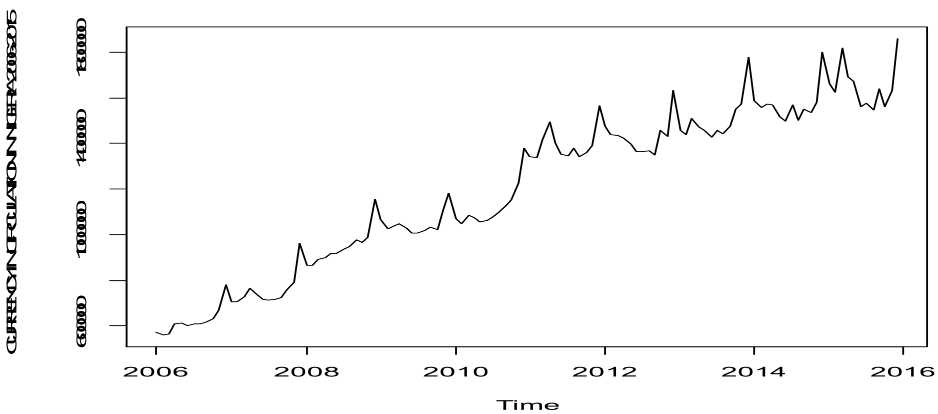

Figure 1: A plot of Currencies in circulation in Nigeria (2006-2015)

From Figure 1 the pattern exhibits trend behaviour because the trend pattern exist when the data generally exhibit random fluctuations, a time series may also show gradual shifts or movement to relatively higher or lower over a longer period of time. If a time series plot exhibits this type of behaviour, we say that trend pattern exists. Therefore, figure 2 delineate that trend pattern exist

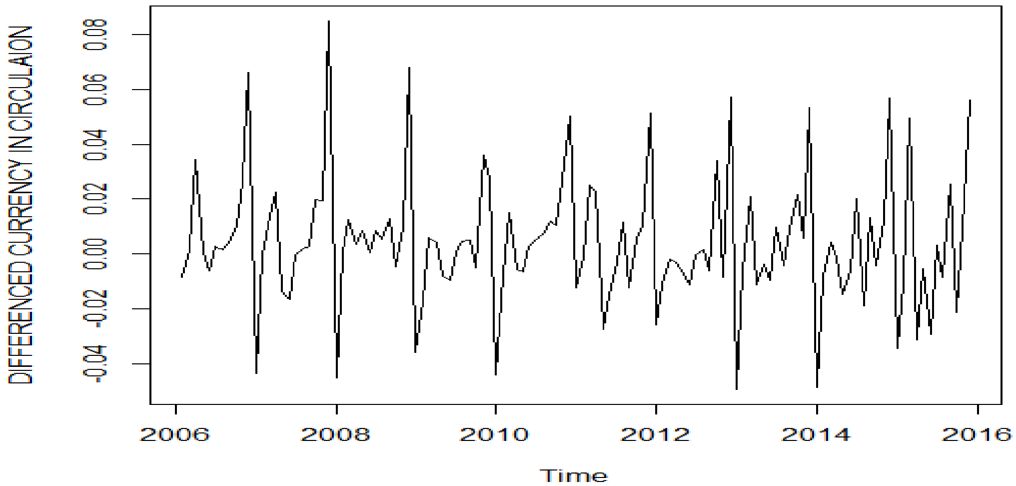

Figure 2: A plot of the log difference of currencies in circulation (2006-2015)

The above figure depicts that the series is stationary in terms of mean and variance that is, this portrait that there is no any variability between the pattern of the series. Decision rule: Since the KPSS of the differenced currency in circulation is (0.049) < critical value (0.120) at 10% level of significant, this implies that the series is stationary at integrated of order one. A time series is said to be stationary if its mean and variance are constant over time and the value of the covariance between the two time period depends only on the distance or gap or lag between the two times periods and not the actual time at which the covariance is computed. In short, if a time series is stationary, its mean, variance and auto covariance (at various lags) remain the same no matter at what point we measure them.

Unit Root Test

Table 1: KPSS test statistics and critical values

Test statistics	1% critical values	5% critical values	10% critical value	P value
0.15046	0.216	0.148	0.120	0.049

Table l indicates that the test statistics 0.15046 > 0.120 we reject H_0 and conclude that the series is not stationary and therefore integrated of order one makes the series stationary.

The Augmented Dickey Fuller (ADF) Test

Table 2: Augmented Dickey Fuller with Constant

estimated value of (a – 1)	**-0.044213**
test statistic: tau_c(1)	-1.98082
asymptotic p-value	0.2956
1st-order autocorrelation coeff. for e	-0.005
lagged differences: F(24, 69)	7.354 [0.0000]

Since the asymptotic p- value 0.2956 > critical region 0.05 we do not reject the null hypothesis H_0 and therefore conclude that the series is stationary,

Table 3: Augmented Dickey Fuller with Constant and Trend

estimated value of (a – 1)	**-0.191778**
test statistic: tau_ct(1)	-1.98082
asymptotic p-value	0.2956
1st-order autocorrelation coeff. for e	-0.077
lagged differences: F(12, 92)	10.589 [0.0000]

Since the asymptotic p- value 0.2956 > critical region 0.05 we accept the null hypothesis H_0 and therefore conclude that the series is stationary.

Test for Autocorrelation of Currencies in Circulation

Before we can determine the estimate of the parameter for the time series we first of all identify the order of the model, the autocorrelation function (ACF) and partial autocorrelation function (PACF) factor plot which can help to identify the pattern in the stationary series of currency in circulation, the idea is to identify the presence of AR and MA components in the residuals

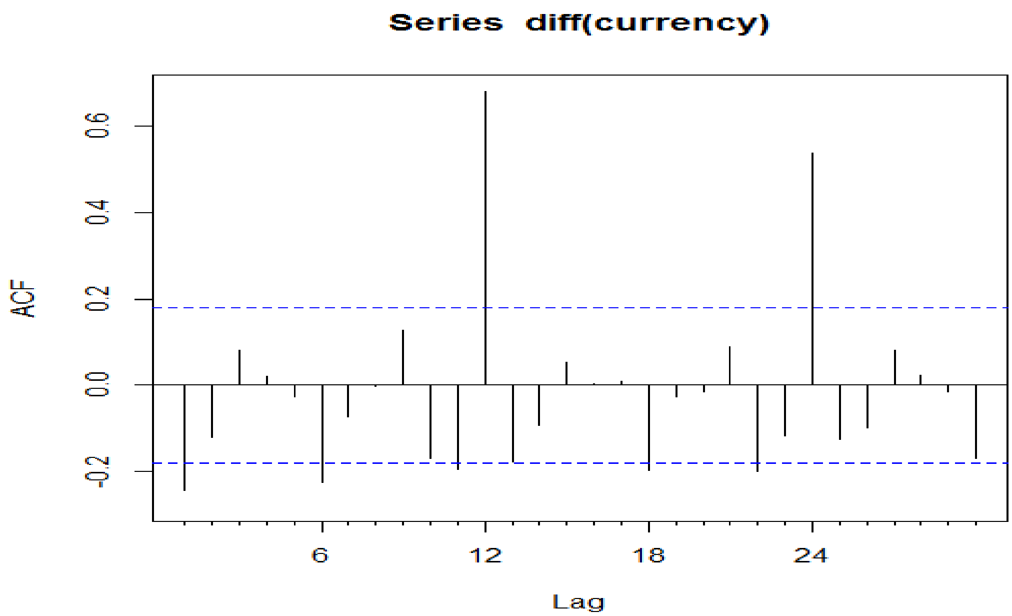

Figure 3: Autocorrelation Function (ACF) of currencies in circulation

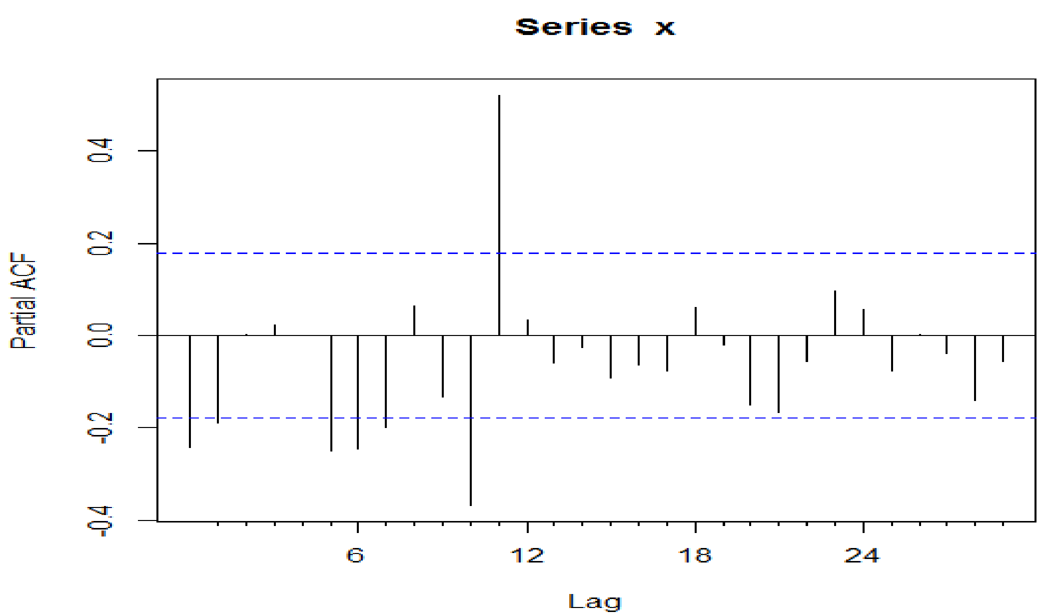

Figure 4: partial Autocorrelation Function (PACF) of currencies in circulation

Figure 3 and 4 shows that, since there are spikes in the plots outside the significant region we conclude that the residuals are not random. This implies that, there is information available in residuals to be extracted by AR and MA models, 12 ARIMA (p, d, q) models were examine from which, based on the model selection criteria of AIC and BIC, the appropriate model is selected.

Table 5: Postulated models and evaluation

MODEL ARIMA (p d q)	AIC	BIC
ARIMA (1,1,0)	2580,13	2588.15
ARIMA (1,1,1)	3001.3	3009.64
ARIMA (1,1,2)	3002.16	3013.28
ARIMA (1,1,3)	3002.93	3016.83
ARIMA (2,1,1)	3001.76	3012.88
ARIMA (2,1,2)	2989.82	3003.72
ARIMA (2,1,3)	2991.72	3007.85
ARIMA (3,1,1)	3003.59	3017.48
ARIMA (3,1,2)	2982.79	2999.46
ARIMA (3,1,3)	2984.79	3004.24
ARIMA (5,0,0)	3036.41	3055.95
ARIMA (5,1,1)	3006.07	3025.53

In the case of ARIMA (p,d,q) models and evaluation ARIMA (5,0,0) has the highest value of AIC and BIC followed by ARIMA (5,1,1), ARIMA (3,1,1), ARIMA (1,1,3), ARIMA (1,1,2), ARIMA (2,1,1), ARIMA (1,1,1),ARIMA (2,1,3), ARIMA (2,1,2), ARIMA(3,1,3), ARIMA (3,1,2) and lastly ARIMA (1, 1, 0) which has the minimum value of AIC and BIC, therefore it served as the best model.

Table 6: Parameter estimate of the fitted ARIMA model

Δ	**10462.1**	**5157.89**	**2.0284**	**0.0425**
α_1	**−0.263713**	**0.091ε5413**	**−2.8808**	**0.0040**

Table 7: Parameter estimate and criterion

Mean dependent var	10799.95	S.D. dependent var	73698.99
Mean of innovations	−47.90789	S.D. of innovations	70938.19
Log-likelihood	−47.90789	Akaike criterion	3002.136
Schwarz criterion	3010.473	Hannan-Quinn	3005.521

From Table 6 and 7, Schwarz information criterion has the highest value followed by Hannan Quinn and Akaike information has the least value that is why ARIMA (1,1,0) served as the best model with minimum value of AIC and BIC.

Table 8: A Forecasts of Currencies in circulation (2016 – 2018)

MONTHS	YEARS		
	2016	**2017**	**2018**
January	1811898.18	1949235.24	2074780.62
February	1843794.04	1959697.36	2085242.74
March	1843794.04	1970159.47	2095704.85
April	1855292.46	1980621.59	2106166.97
May	1865481.29	1991083.70	2116629.08
June	1876015.48	2001545.82	2127091.20
July	1886458.59	2012007.93	2137553.31
August	1896925.71	2022470.05	2148015.43
September	1907386.51	2032932.16	2158477.54
October	1917848.97	2043394.28	2168939.66
November	1928310.99	2053856.39	2179401.77
December	1938773.13	2064318.51	2189863.89

Table 8 shows the forecast of currency in circulation from 2016 to 2018 where by the year 2018 has the highest currencies in circulation. Which indicate that Nigeria will expect high currencies in circulation in the year 2018.

Conclusion

This research investigates monthly currencies in circulation in Nigeria from 2006 to 2015. The aim is to Model and forecast currencies in circulation in Nigeria with different types of ARIMA family models. The result from all the models was obtained and ARIMA (1, 1, 0) served as the best model among the 12 ARIMA models. And also the research proposed the ARIMA model for currencies in circulation (CIC), the model evaluates the performance and forecast three year observation (i.e. from 2016 to 2018) of the process and the results shows a significant increase of currencies in circulation in Nigeria from 2016 to 2018 experience to never has. The research can serve as step to observe the Currencies in circulation in Nigeria. Specified period data can be tested by future researchers by developing new and more models to capture the effect and prediction of the currency behaviour circulated in Nigeria.

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